

If you have some rational expressions (polynomial fractions) you can find a common denominator and add them together

$$\frac{1}{x+3} - \frac{2}{(x-1)} + \frac{x+1}{x^2+1}$$

$$= \frac{1(x-1)(x^2+1)}{(x+3)(x-1)(x^2+1)} - \frac{2}{(x-1)}$$

$$\frac{1}{x+3} - \frac{2}{x-1} + \frac{x+1}{x^2+1}$$

$$= \frac{(x-1)(x^2+1)}{(x-1)(x+3)(x^2+1)} + \frac{-2(x+3)(x^2+1)}{(x-1)(x+3)(x^2+1)} + \frac{(x+1)(x-1)(x+3)}{(x-1)(x+3)(x^2+1)}$$

$$= \frac{x^3 - x^2 + x - 1}{(x-1)(x+3)(x^2+1)} + \frac{-2x^3 - 6x^2 - 2x - 6}{(x-1)(x+3)(x^2+1)} + \frac{x^3 + 3x^2 - x - 3}{(x-1)(x+3)(x^2+1)}$$

$$= \frac{-4x^2 - 2x - 10}{(x-1)(x+3)(x^2+1)}$$

If you were integrating, it would be easier to integrate the smaller expressions than the larger one:

$$\int \frac{1}{x+3} - \frac{2}{x-1} + \frac{x+1}{x^2+1} dx = \int \frac{-4x^2 - 2x - 10}{(x-1)(x+3)(x^2+1)} dx$$

Easier

Harder

Is there a way to go backwards from the harder, combined, version to the easier, expanded, version?

Yes! But first you need to know what you're looking for.

For instance:

$$\frac{-4x^2 - 2x - 10}{(x-1)(x+3)(x^2+1)} = \frac{A}{x+3} - \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$$

Here's how to get the right form to look for...

Step 1:

Is the degree of the numerator **smaller** than the degree of the denominator?

$$\frac{1}{x+3} - \frac{2}{x-1} + \frac{x+1}{x^2+1} = \frac{-4x^2 - 2x - 10}{(x-1)(x+3)(x^2+1)}$$

← deg 2  
← deg 4

$$\frac{x^2+3x+5}{(x+2)(x-2)}$$

← deg 2 } No  
← deg 2 }

$$\frac{3x+7}{(x+5)(x-1)}$$

← deg 1 } yes  
← deg 2 }

$$\frac{x^3}{x^2-5x+6}$$

← deg 3 } No  
← deg 2 }

$$\frac{x^3}{(x+4)(x-3)(2x+1)}$$

← deg 3 } yes  
← deg 4 }

If the degree is smaller, then go on to step 2. If it's **not smaller**, then you need to divide first to find the polynomial part. Then you can do step 2 with what's left after dividing.

$$\frac{x^2+3x+5}{(x+2)(x-2)}$$

$$x^2-4 \overline{) x^2+3x+5}$$

$$\underline{-x^2-0x-4}$$

$$0+3x+9$$

$$= 1 + \frac{3x+9}{(x+2)(x-2)}$$

$$\frac{8}{5} \quad 5 \overline{) 8}$$

$$\underline{5}$$

$$3$$

$$\frac{3}{5}$$

$$\frac{x^3}{x^2-5x+6}$$

$$x^2-5x+6 \overline{) x^3 \quad 0 \quad 0}$$

$$\underline{-x^3+5x^2-6x}$$

$$0+5x^2-6x \quad 0$$

$$\underline{-5x^2+25x-30}$$

$$0+19x-30$$

$$x+5 + \frac{19x-30}{x^2-5x+6}$$

Step 2: Figure out the format for the expanded version:

$$\frac{-4x^2 - 2x - 10}{(x-1)(x+3)(x^2+1)} = \frac{1}{x+3} - \frac{2}{x-1} + \frac{x+1}{x^2+1}$$

$$\frac{3x+9}{(x+2)(x-2)} = \frac{A}{(x+2)} + \frac{B}{(x-2)}$$

$$\frac{3x+7}{(x+5)(x-1)} = \frac{A}{(x+5)} + \frac{B}{(x-1)}$$

$$\frac{19x-30}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2}$$

$(x-3)(x-2)$

$$\frac{x^3}{(x^2+4)(x-3)(2x+1)} = \frac{Cx+D}{x^2+4} + \frac{A}{x-3} + \frac{B}{2x+1}$$

$$\frac{3x^2 + 5x + 7}{(x-3)(2x+1)(x+5)} = \frac{A}{x-3} + \frac{B}{2x+1} + \frac{C}{x+5}$$

$$\frac{3}{(x^2 + 2)(x-3)(2x+5)} = \frac{Ax+B}{x^2 + 2} + \frac{C}{x-3} + \frac{D}{2x+5}$$

$$\frac{7x+4}{(x+1)^2(x-3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3}$$

$$\frac{x+2}{(x^2+5)(x-3)^3} = \frac{Ax+B}{x^2+5} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{F}{(x-3)^3}$$

