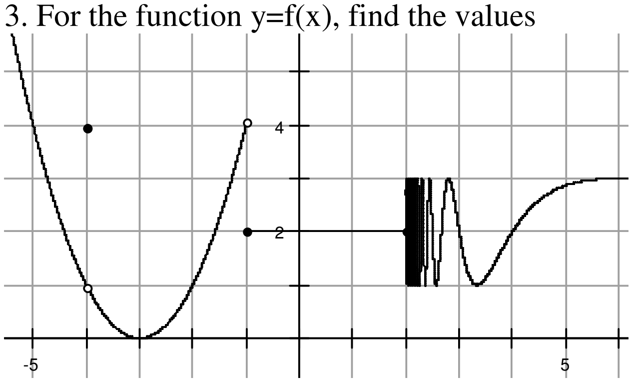
Math 166: Calculus I Review problems for exam 1

1. Graph each of the function: $y = \dfrac{x^3 + x^2 - 6x}{x^2 - 3x + 2}$

2. Sketch the graph of a rational function that has
- a root of multiplicity 2 at x=2,
- a root of multiplicity 1 at x=-3,
- a vertical asymptote with mult. 1 at x=-1
- a vertical asymptote with mult. 2 at x=4
- a missing point at (5, 1)

3. For the function y=f(x), find the values



a. $\lim_{x\to -4} f(x)$ b. $f(-4)$ c. $\lim_{x\to -1^-} f(x)$ d. $\lim_{x\to -1^+} f(x)$
e. $\lim_{x\to -1} f(x)$ f. $f(-1)$ g. $\lim_{x\to 0} f(x)$ h. $\lim_{x\to 2^-} f(x)$
i. $\lim_{x\to 2^+} f(x)$ j. $\lim_{x\to 2} f(x)$ k. $f(2)$
l. Tell where $f(x)$ is discontinuous, and why.
m. Tell the intervals of continuity for $f(x)$

4. For this function:

$$f(x) = \begin{cases} x+2 & x < -1 \\ x^2 - 1 & -1 \le x < 2 \\ 3 & 2 < x \end{cases}$$

a. Sketch the graph
b. Find the left and right and normal limits at x=–1 and 2
c. If it is not continuous at x=-1 and/or 2, tell why it is not continuous

5. Use the squeeze theorem to find the limit of $\lim_{x\to 0}(x^2(\sin\left(\dfrac{2}{x}\right) - 1))$. Justify your answer.

6. Find all of the values of b so that the specified function is continuous:

a. $y = \begin{cases} 4 - x^2 & x < 2 \\ x + b & x \ge 2 \end{cases}$

b. $y = \begin{cases} b^2x^2 - 3x + 3 & x < 1 \\ bx + 2 & x \ge 1 \end{cases}$

7. Find the derivative $f'(a)$ for each of these functions by using the limit definition/formula for the derivative.

a. $f(x) = 3x^2 - x + 1$

b. $y = \dfrac{2}{x-3}$

c. $f(x) = 3\sqrt{x-2}$

8. Use your results from #7 to help you find the equation of the tangent line:

a. to $f(x) = 3x^2 - x + 1$ when x= 2

b. to $y = \dfrac{2}{x-3}$ when x=-1

c. to $f(x) = 3\sqrt{x-2}$ when x=3

9. Ellen made 3 errors in using limits to find the derivative of $y = x^2 - x$ (one error is repeated multiple times). Find and fix her errors:

$(x, x^2 - x) \quad (x+h, (x+h)^2 - x + h)$

$$m = \frac{(x+h)^2 - x + h - (x^2 - x)}{x + h - x}$$

$$\lim_{h\to 0} \frac{x^2 + h^2 - x + h - x^2 + x}{h} = \frac{h^2 + h}{h}$$

$$= \frac{h(h+1)}{h} = \lim_{h\to 0} h + 1 = 1$$