**Multiplication facts**

**Goals for multiplication facts?**

* Recall from memory all (most) of the multiplication facts up through 10 × 10 (12 × 12)
* Given a product, recall from memory all (most) of the multiplication facts that give that product

**What are we doing already**?

* Early lessons in multiplication
* Assigning multiplication problems (multi-digit)
* Flash cards
* Practice sheets for multiplication facts
  + All 100 (144) facts
  + 10 (20) facts at a time
* Lessons…?

**The problem**: memorizing is hard, so we need to find patterns and tricks and we need to be systematic when and how we’re learning.

**Goals for scaffolding**: there are only so many non-sense rhymes you can keep straight, so save them for the really hard facts, and teach as many facts as possible by finding math patterns to help with remembering and figuring out.

**Invitation**: Try some multiplication fact teaching strategies with me that I’ve found helpful in working with 3rd-6th grade students

**Caveat**--this is for the students in the middle:

* some students don’t need this program—they’ll pick up the multiplication facts without this much work (and probably have already)
* some students still won’t learn the multiplication facts (usually students with a math learning disability), and later (maybe January) we can discuss things we do to help those students (you’re probably doing some of those things already)

**Overview**:

* Break the facts into groups and learn one set at a time
* Start by practicing ways to think about and figure out the facts in that group
  + Each group has a good strategy you can learn: a strategy that’s faster than skip counting, and much faster than counting by 1’s.
  + Practice the strategy: what to do to get the answer.
* Practice that set of facts with tasks that are not timed flash cards
* Follow untimed practice with a very short timed, but low-stakes quizzes
* Use quiz times to determine when to move on to the next set of facts.
* After a set of facts is mastered, continue review, but less frequently.

**The flow**:

* Learn × 0 and × 1 facts (or verify that students already know these)
* Learn × 10 facts (or verify that students already know these)
* Learn × 2 facts by thinking doubles eg. 6 × 2 = 6+6 (most students will know most but not all of these. These are a building block set of facts, so overlearn them)
* Learn × 3 facts by thinking double and add, eg. 6 × 3 = (6+6)+6 (these will take a while to master. It’s OK to move on even if some students are still working on some facts)
* Learn × 4 facts by thinking double and double the double, eg. 6 × 4 = (6+6) + (6+6) = double 12. (like 3’s, it’s OK to move on before these are overlearned.
* Learn 5 × facts by putting 5’s together to make 10s and by looking for patterns in the multiplication table (especially for even × 5). Overlearn these, and spend some time reviewing the facts so far before moving on—we need the 5’s to be solid.
* Learn 9 × facts by finding patterns. I like the pattern where you go back 1 for the tens digit and then add up to 9 for the ones digit, eg. 7 × 9 = 63 (7-1 = 6, so 60 and 6+3=9 so 63). This is the same pattern as the finger pattern of course (ask someone to show you). Kids usually like the finger pattern at first, and then later it’s helpful to realize that you’re always going back one for the tens digit as a memory aid that’s faster than getting out your fingers. It’s more important that they be self-reliant than be fast, though, so fingers are fine.
* Learn × 6 facts by doing × 5 and adding on, eg. 8 × 6 = (8 × 5) +8. This works really well for 6 × 6 and 8 × 6, and it a little awkward for 7× 6 (they should already know everything else!). If you know a good rhyme for 7 × 6 you can teach it now, or you can just keep practicing.
* Learn × 7 facts by doing × 5 and × 2 and adding together, eg. 7×7=(7 × 5) + (7 × 2). This works well for 6×7 and 7× 7 and even 7× 8. The strategy can be done efficiently (no carrying in the addition), but it’s harder to get used to, so teaching rhymes are OK here too.
* The only fact left to learn is 8× 8. Doing 8× 5 + 8× 3 works well, but now it’s time to sit down and practice the new 6’s 7’s and 8’s. These are the hardest for everyone to remember but there’s really only 4-6 of them, so if you pick one of those as your fact for the day and randomly ask it during transition times, you can make some headway like that too.

**Teaching the × 2 facts**

Notes on pre-assessment p. 4

Assess to find out which students already know the × 2 facts fluently (optional)

Option 1: × 2 interview outline p. 5

Option 2: × 2 student self-report p. 6

Prerequisite knowledge: knowing how to show on your fingers the numbers 1-10 in the standard (American) format with automaticity.

Assess (for students who do not know the × 2 facts fluently)

Finger flash interview p. 5

Finger representations

Notes on teaching finger representations p. 7

Lesson 1 p. 7

Lesson 2 p. 8

Matching game p. 9-10

Flash cards p. 11-15

Notes on teaching × 2 strategy and practice

× 2 multiplication facts

Notes on teaching × 2 strategy and practice p. 16

Lesson 1: strategy and memory game task p. 17

Lesson 2: review, commutative law, Bump game and flash cards p. 18

Strategy Reminder sheet p. 19

Matching game for strategy p. 20-27

Matching game for product with reminders p. 28-35

Flash cards with reminders p. 36-39

Flash cards, no reminders (optional) p. 40-43

Memory (matching game) cards, no reminders (optional) p. 44-45

Worksheets for independent practice (2 per sheet) p. 46-50

Quick Quizzes (timed quiz sheets, 4 per sheet) p. 51-54

Bump game board and instructions p. 55

Notes on pre-assessment

If one were working with students individually, then one would, of course, want to do a pre-assessment to give students work at the right level for them—students who are already fluent at doubling numbers don’t need to do this work. With a whole class, one could assign the whole class to do the work, but I would suggest breaking students into groups if possible, where students who are fluent will do only a small number of timed assessments, and students who are not fluent will do more of the lessons and activities.

* You probably know students in your class who seem to be proficient with all of the multiplication facts, so you probably don’t need to assess their knowledge—you can safely assume they will need only a minimal number of lessons if any.
* You may know some students in your class who seem to struggle with all of the math facts (counting is their primary strategy for everything). You probably don’t need to do the doubles assessment with them, but you should find out if they can fluently show the numbers 1-10 on their fingers (without counting fingers). If they’re going to mix anything up it will be 7 and 8, but they may be counting to put up that many fingers.
* Then there are students in the middle, where you’re not sure, and you’re going to need to find out if they have the doubles memorized or not.

In doing the pre-assessment, I predict that all of your students will know 0+0, 1+1, 2+2, 3+3, 4+4, 5+5 and 10+10, and a lot of the students won’t know 7+7 and 8+8 without counting. (But some will know all of the doubles)

**Why pre-assess doubles with addition?** It’s going to be more efficient soon if we’re thinking of × 2 as doubles: 6×2=6+6, not skip counting by 2’s. We want to know who already knows those addition facts, even if they haven’t internalized it as *the right way to multiply by 2*, so we’re assessing that skill first.

**Why finger patterns**? For children who are stopping to figure out the doubles (× 2) facts, especially if they are counting, then knowing the finger patterns for the numbers 1-10 is an important pre-requisite to using this way of figuring out doubles quickly. The useful thing about the standard finger patterns is that the numbers 6-9 are represented using 1 whole hand (5) and some more fingers on the other hand, so it breaks down those numbers into 5 and some more—this gives a fast, visual way to figure out the harder doubles: 6×2, 7×2, 8×2 and 9×2.

**Why on the same page?** The interviews are on the same page so that you can go straight from the doubles interview to the finger patterns interview for children who don’t know all of the double facts.

**The doubles interview should be done with children individually** since you are listening for their pace and where they are hesitating.

**The finger flash interview *could be done* with a small group.** It’s possible to keep track of who was fast and automatic and who hesitated and counted with 3-4 students at a time.

See the Notes on Teaching page for using the pre-assessment data.

**Doubles pre-assessment interview**

Ask these doubles problems verbally. Record for each problem: √ (answered fast) S (answered slow) M (medium speed)

After the interview, ask the child about the problems they answered more slowly (if any). For example:

“It took you a little longer to answer 7+7—did you remember that one, or did you figure it out? How did you figure it out--did you count or did you do something else?”

Make notes on their strategies.

name:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 4+4 | 10+10 | 7+7 | 2+2 | 6+6 | 1+1 | 5+5 | 9+9 | 3+3 | 8+8 |
|  |  |  |  |  |  |  |  |  |  |

**Finger Flash interview**

Look for the following behaviors as children show each number on their fingers:

* Is it quick and automatic?
* Do they stop to count?
* Do they mix up 7 and 8?

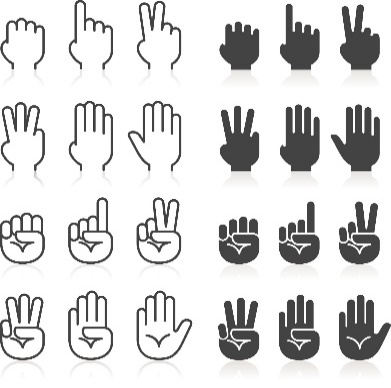
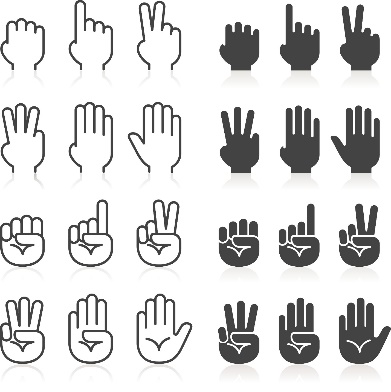
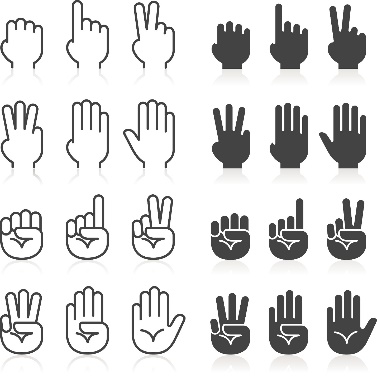
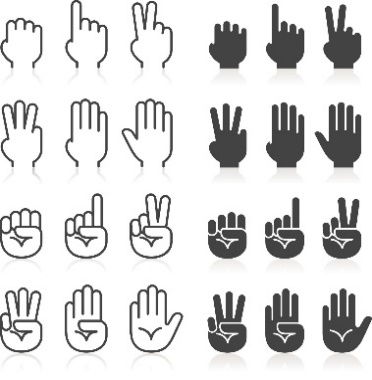
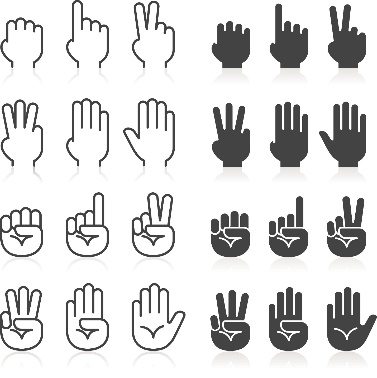
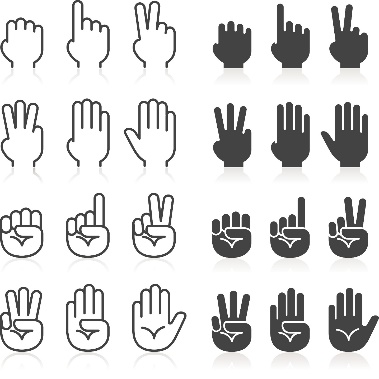
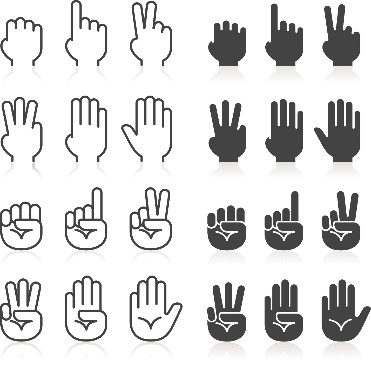
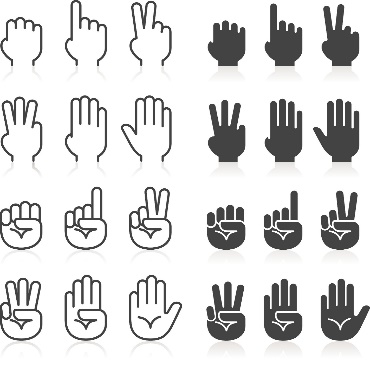
Note if students count out the numbers to show them on their fingers, or if they mix up 7 and 8 (show 8 fingers for 7 or vice versa).

Ask children to quickly show each number with their fingers as you call it out:

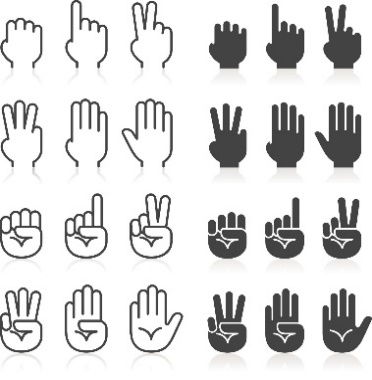
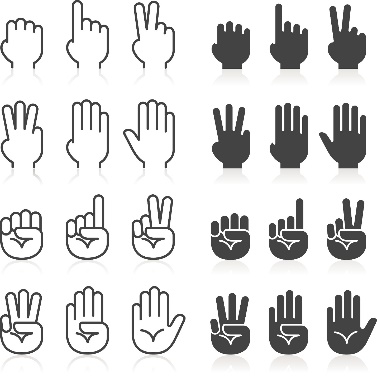
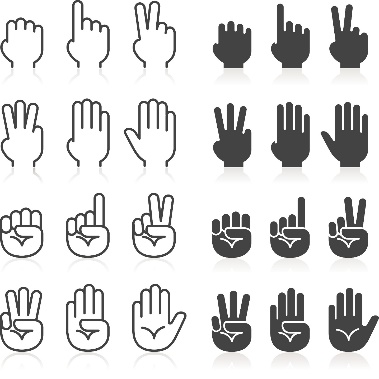
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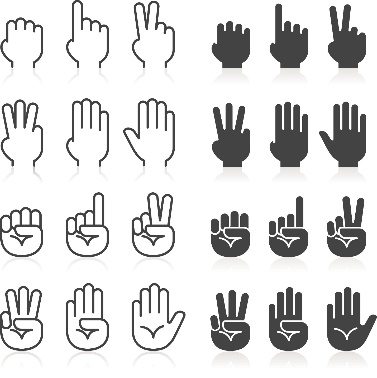
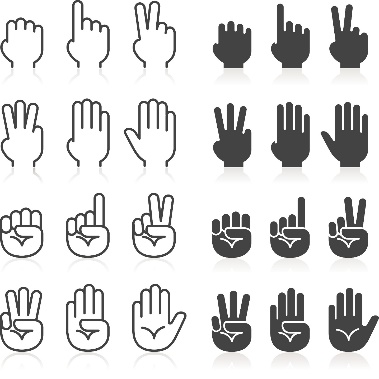
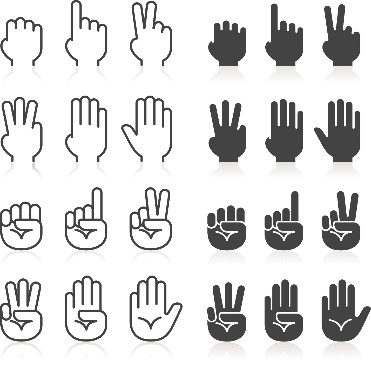
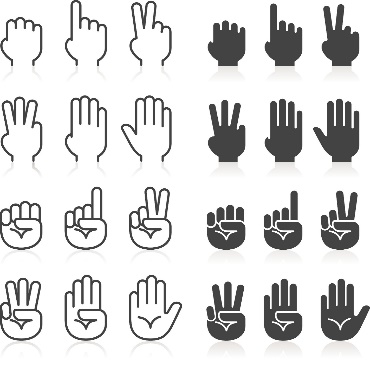
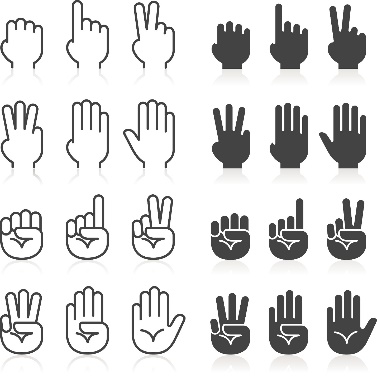
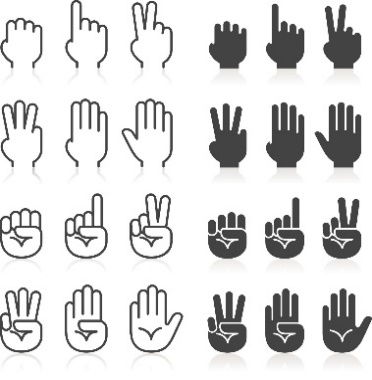
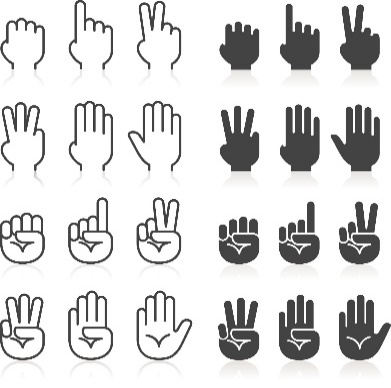
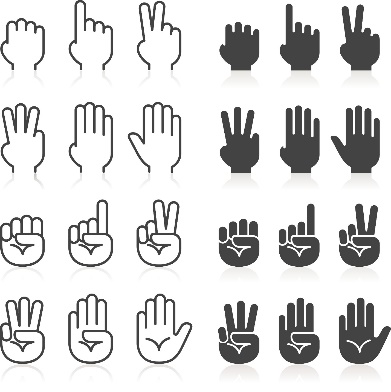
(You’re just watching what they do for 7, 8, 6, 9 and 7 again—the other numbers are just in there to set a regular pace)

* Do they show the numbers 6-9 in the usual way (with all 5 fingers up on one hand)?



If a child is showing the numbers 6-9 correctly but in another way, then do a backwards interview, and ask them to say the number quickly for each of your finger flashes:





**Doubles Pre-Test**

Answer each question. Check the box to say whether you remembered the answer right away or if you stopped and figured it out

|  |  |  |
| --- | --- | --- |
|  | Remembered | Figured out |
| 3+3 = ­­­\_\_\_\_\_ |  |  |
| 10+10 =­­­\_\_\_\_\_ |  |  |
| 6+6 =­­­\_\_\_\_\_ |  |  |
| 4+4 =­­­\_\_\_\_\_ |  |  |
| 9+9 =­­­\_\_\_\_\_ |  |  |
| 5+5 =­­­\_\_\_\_\_ |  |  |
| 1+1 =­­­\_\_\_\_\_ |  |  |
| 8+8 =­­­\_\_\_\_\_ |  |  |
| 2+2 =­­­\_\_\_\_\_ |  |  |
| 0+0 =­­­\_\_\_\_\_ |  |  |
| 7+7 =­­­\_\_\_\_\_ |  |  |

**Notes on Teaching Finger Representations**

**Using the pre-assessment:**

Children who need to count to figure out finger representations for the numbers 6-9 should do these lessons.

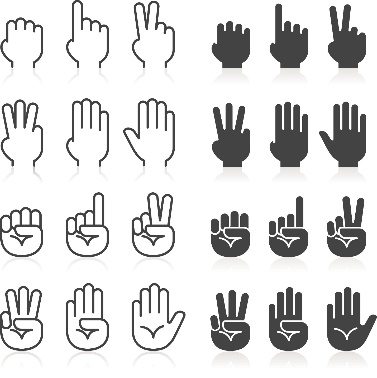
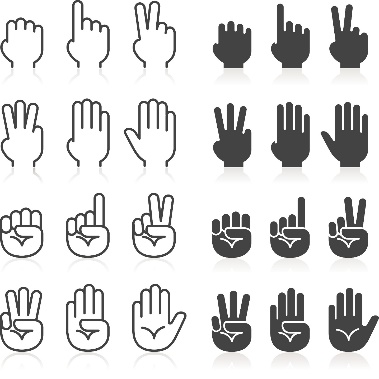
Children who use non-standard representations *and* who need to count to figure out how many fingers are being shown using a standard finger-flash should do these lessons

Children who automatically can show finger flashes in the standard way *or* automatically know how many fingers are shown in the standard way do *not* need these lessons

**Timing: 1 week.** Students need to be good at this before you start the doubles lessons, but they don’t have to be perfect. Practicing for a week will get them ready to think about doubles this way.

**Lesson 1 [needs finger flash matching game cards]:**

*Explain why this is useful:* “We’re going to practice showing the numbers between 0 and 10 on our fingers. This is going to help us figure out how to double numbers. If I show 6 on my fingers like this:



I’m showing 5 on one hand and 1 on the other to make 6. Who knows what 5+5 is? …. Yes, 10, and 10 is a really easy number to add with, so having 5 on one hand and 1 on the other is going to help us add faster.

*Introduce 7:*Now show me 7 fingers [wait]. Yes. That’s 5 on one hand and 2 on the other.

*Practice:* Show me 6 fingers, now 7 again, and 5, and 7, and 3, and 6 and 1 and 7

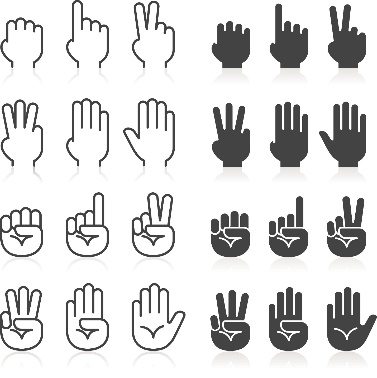
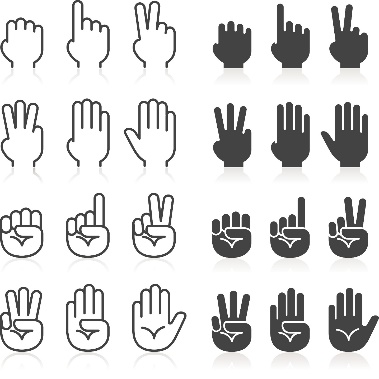
*Introduce 9:* Who can show me 9 fingers [wait] Yes, 9 is 5 on one hand and 4 on the other. It’s one less than 10.

*Practice:* Now show me 7 fingers, and now 9, and 7 and 9 and 7 and 3 and 7 and 9 and 2 and 5 and 9 and 6 and 4 and 6 and 9 and 7

*Introduce 8:* Our last number is 8. Show me 8 fingers. [wait] Yes, 8 is 5 on one hand and 3 on the other.

*Practice:* Show 8 and 7 and 9 and 8 and 6 and 8.

*Assign:* These cards are a matching game--mix them up face down, and then take turns looking for matching cards. Two cards match if they show the same number, so the number 6 and the finger pattern for 6 are a match:



6

Assign students to play the matching game at least twice each day for at least 2 days

**Lesson 2:**

*Review:* Show me 7 fingers. Show me 9 fingers. 6 fingers. 8 fingers. 7…5….8…2…9…3…4…7…10…6

*Assign:* These cards are flash cards, but I want you to do them in partners\*

One partner should mix up the cards and hold them up one at a time so it shows the number, and the other partner should hold up that number of fingers.

Then turn them over and show the fingers, and the other partner should say the number that goes with the hand picture. \*\*

When you’ve done all the numbers and all the pictures, then switch jobs and the second person will show the cards and the first person will do the answers.

\*If you have an odd number of students doing this, you can let them work in groups of 3. They can do it in the same way a 2-partner group would, and each person should get to do the answers and hold up the cards at least once, or (if you think they can handle the responsibility to play quietly) they can do it as a game: one person hold up the card and the other two try to answer as quickly as possible.

\*\*If it’s going to be too loud for your classroom for them to say the numbers out loud, then they can do the finger-flash answers, going through the cards twice instead.

[play one of the two games, or practice individually with flash cards for several days—at least 4 days practice total]

**Notes on teaching × 2 strategy and practice**

**2’s should be learned as doubles**—skip counting by 2’s works, but it’s not the thinking strategy that’s going to help most when we move on to 3’s and 4’s, so the best teaching and learning strategy is to get good at thinking about, figuring out, and memorizing (in that order) doubles.

**Children should have learned or be learning about the commutative law** in a meaningful (visual/concrete) way prior to this, but during facts discussions, the default should be to treat the commutative law as something they already know and should remember (revisiting earlier commutative law lessons only if students are uncomfortable with switching the order of factors)

It’s usually a safe assumption that children already know 2+2, 3+3, 4+4, 5+5, 10+10, so **these lessons focus on efficiently figuring out 6×2, 7×2, 8×2 and 9×2**.

After children have practiced using the doubles finger-flash strategy for figuring out doubles, students should be practicing the doubles facts daily.

* Practicing is not timed, and repeats the same 10 facts over and over in a random order.
* At least for the first several days, students should have the **Doubles reminder sheet** during practice time to reinforce using the doubles strategy for problems that are not instantly remembered.
* There are several options for practice:

Cards for playing a memory game

Materials for a board game “Bump”

Flash cards

Worksheets (see notes on order of worksheets)

* Students do not need to do all of the practice tasks available
* Immediately after a practice task or game (any except memory: introductory doubles) is the best time for students to take a timed quiz. If you have computers or tablets available, there are a lot of good free apps that do this: choose one that has a very simple format (just the numbers, no fancy pictures), and will time students on basic ten × 2 facts. Alternately, there are several versions of such quizzes included, and students can time each other with a stopwatch or just a clock with a second hand.
* When students have passed the timed quizzes several times with times at about 30 seconds, then they have probably done enough practice. After that, they should just do the timed test occasionally, with no extra practice unless they start missing problems or taking longer.

**Using the pre-assessments:**

* Students who have to stop to figure out some of the doubles facts should do these lessons (after mastering finger-number patterns).
* Students who are fluent with the doubles facts, and you believe to be fluent in all or almost all of the other multiplication facts can be excused from these lessons entirely.
* For students who are fluent with the doubles but not all of the multiplication facts, I suggest that they observe the first lesson to understand the strategy the class will be using, and should take at least 4 timed quizzes, spaced out over the two weeks to cement the facts in their memory.

**Lessons and practice for × 2 facts should last approximately 2 weeks**

**Lesson 1: Teaching the strategy and the matching games [copy the two different doubles matching games onto different colored paper]**

*Introduce the goal:* This week we’re going to practice the × 2 facts.

*Introduce the idea that × 2 means doubles:*

* Remember that 5×2 means 5 taken 2 times, so we want the sum of two 5’s.
* If I ask you what 5+5 is, you would all say [gesture for everyone to answer together].
* So if I ask what 5× 2 is, you should all say [prompt for everyone to answer]
* And if I ask you what 4× 2 is, you would all say [prompt]
* And if I ask what 10× 2 is, you would all say [prompt]

*Introduce the strategy of finger flashes for the harder problems:*

* Now a problem that might take a little longer to figure out would be 7× 2, so I have something I want you to try.
* This is a partner strategy so I want [student 1] and [student 2] to stand up
* [*to the students*] I want you to both hold up 7 fingers, for the 7+7, and I want you to put your hands next to each other that have 5 fingers up, and I want you to put the hands next to each other that have 2 fingers up.
* [*to the class*] If you look at these two hands, they’re showing 5 and 5, so how much is that? [call on someone or gesture for everyone to answer]
* If you look at these two hands they’re showing 2 and 2, so how much is that? [call on someone or gesture for everyone to answer]
* And 10+4 is [call on someone or gesture for everyone to answer]
* So that means 7+7 is 14.
* Ask the students to sit down.
* [Flash 7 twice with your hands] 7× 2 is [flash 5 twice] 10 plus [flash 2 twice] 4 is 14.
* Everyone show me 6× 2 with your fingers [flash 6 twice]. [Tell while showing with your hands] So 6× 2 is 10 and 2. Who can tell me what 6× 2 is? [call on a student]
* Everyone show me 8× 2 with your fingers [flash 8 twice].
  + turn to your neighbor and tell them how you can put the 5’s together and the 3’s together to find 8× 2
  + Who would like to show the class how to put the 5’s together and the 3’s together to get 8× 2? [call on a student]

*Demonstrate the matching games:*

* I have two different matching games here. One is green and one is yellow. I’d like you to find a friend and play a matching game with them.
* On the green cards, you are matching the fingers on one card, with the numbers on the other card. [demonstrate a matching pair]
* On the yellow cards you’re matching a problem with the answer [demonstrate a matching pair]
* [If you made enough sets of cards, you can assign them to play a green game first and then yellow. Students can also play the matching game as a solitaire game.]

**Lesson 2: Review the strategy, remind students of the commutative law, and teach the game Bump.**

*Review and commutative law:* [Call on students to answer the questions below. If you have a big group it can be helpful to make a follow up request to another student to re-explain what the first person said to keep the group focused and listening]

* Who can tell me how someone could figure out 8×2 if they didn’t remember it?
* Who can tell me how someone could figure out 6×2 if they didn’t remember it?
* Who can tell me how they would figure out 7×2 if they didn’t remember what it was?
* If I asked you what 2×7 was, would that have the same answer as 7×2 or would it have a different answer? [Follow up by asking why? if you get a simple “yes” answer—make sure the commutative law gets stated or restated clearly: multiplication is the same if you switch the order of the numbers you’re multiplying]
* What is 3×2?
* And what is 2×3?
* What is 10×2?
* And what is 2×10?

*Introduce the game Bump:*

* To play this game you’re going to get a game board [I recommend laminating one], and a bag with game pieces. Each person is going to have 4 counters, and they all need to be the same color. There’s also one 10 sided die in the bag.
* You roll the die and whatever number comes up you need to double it [roll the die] I got the number [6], so I figure out 6× 2 is 12, and I put my marker on 12.
* Invite a student to roll as the other player. Ask them to multiply the number by 2 and cover it with their marker.
* Play a few more rounds—at least until someone rolls a number that is already covered. Explain that if you roll a number that is already covered by your partner, then you get to bump them off: you put your marker on the number, and their marker goes back in their pile. If you roll a number that’s already covered by yourself, then you can bump yourself, or just skip that turn.
* The game ends (and someone wins) when one player plays all of their markers.

Show students the strategy reminder cards, and tell them that they can have a reminder card out during their practice games if they want to.

Give students the options for practice each day this week: playing bump, practicing with flash cards, or working through a set of practice sheets. I find practice to be most effective when it’s done with someone else and when strategy reminders are available, but some students will prefer to work alone (and, perhaps, there are others who you would prefer to have work alone). I find retention from most computerized practice games to be surprisingly low…if you find a really good one, let me know!

*Introduce timed quizzes*: After you do a practice task, you should take a quick quiz. Get a friend to time you. Try to do the quiz without the reminder card.

**Figuring out × 2’s strategy reminder sheet**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 0×2 |  | 4×2 |  | 7×2 |  |
| 1×2 |  | 5×2 |  | 8×2 |  |
| 2×2 |  | 6×2 |  | 9×2 |  |
| 3×2 |  |  |  | 10×2 |  |

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