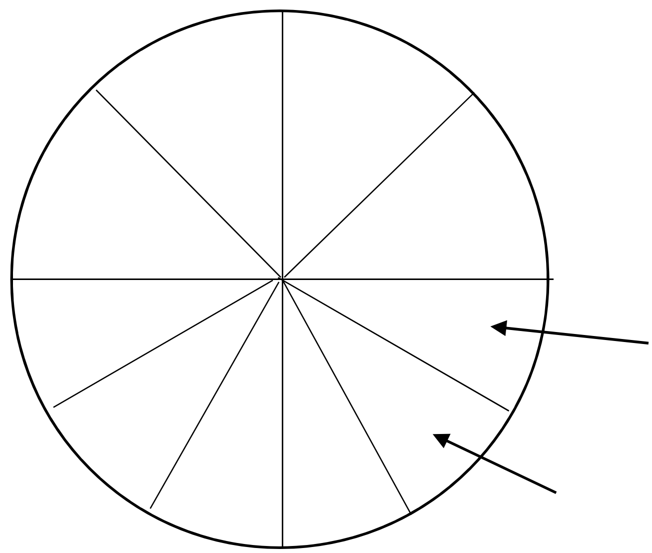
Fractions

Warm up



If I eat the two pizza slices shown, how much of the pizza did I eat?

When my family gets a Papa-Murphy's pizza, I cut it like this—for people who like bigger and people who like smaller pieces of pizza

(I use pizza scissors, btw, which work way better than the circular pizza cutter I used to use)

I usually eat two smaller pieces of pizza. What fraction of a whole pizza am I eating?

Fractions: Some key understandings

Concept 1: Every fraction is a fraction of something

Concept 2—the Standards way of looking at a fraction:

> Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

Concept 3: Fractions can have different names: they have *equivalent* representations.

Concept 4: A fraction tells an amount; it is one number, not two.

Concept 1: Every fraction is a fraction of something—a fraction of a whole unit, whatever your unit is. We kind of do this with multiplication, so it's already in the curriculum in some places, but it's a really important thing to know *and remember* about fractions.

Questions to ask:

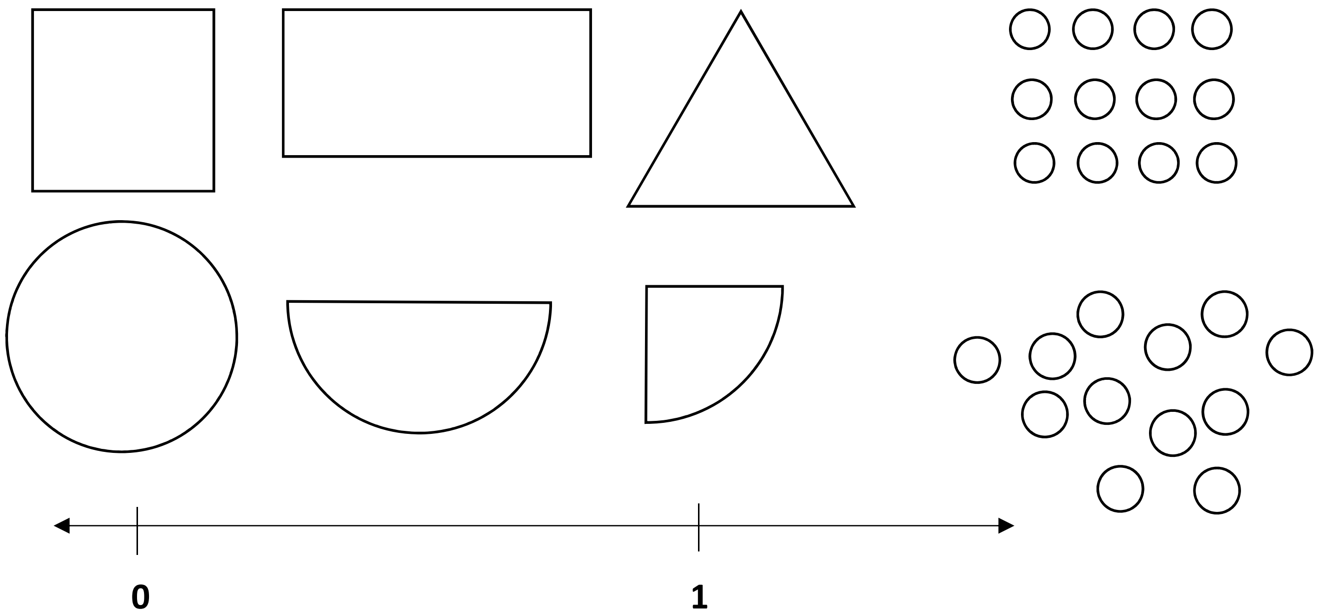
What is [1/2] of a <u>whole</u>? Change both the [fraction] and the <u>whole</u> to make a lot of questions. Do this with standard units: a circle, a square, a foot-length. Make sure some of your units are less standard: use a half-circle as a unit, use a box of crayons as a unit, use a triangle for a unit, and then use a different triangle as a unit, use a group of objects as a unit, use a length of string or strip of paper as a unit, use a length marked on a number line as a unit.

If <u>this shape or amount</u> is [1/2], what is the whole unit? Change the <u>shapes</u> and the [fractions] to make a lot of questions. Make sure that some of the units are common choices (a whole circle, a whole square, an equilateral triangle, a length of a string, a few objects), some others are not (a half-circle, a triangle that's not equilateral, a rectangle that's not a square).

Things you might want to take notes on:

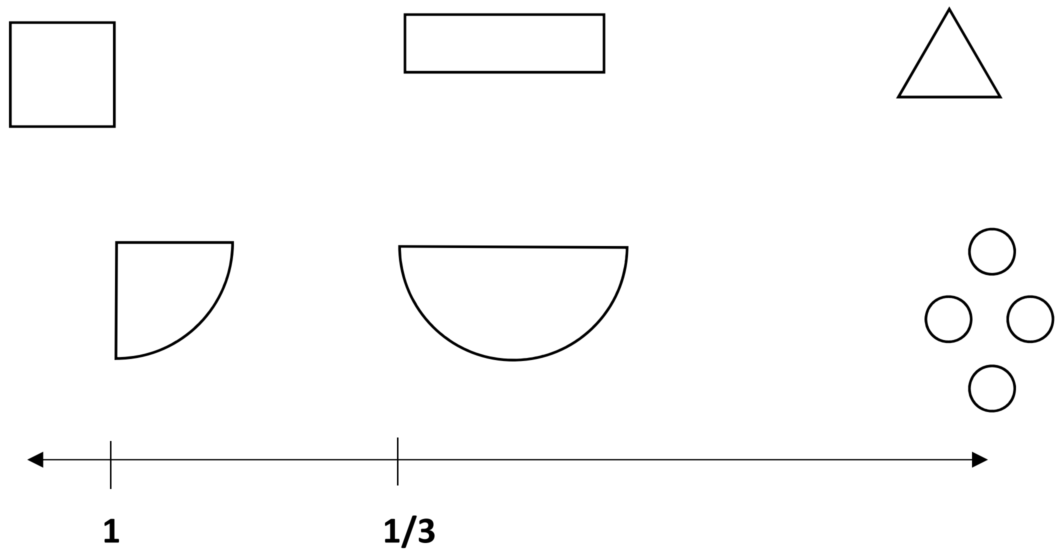
- Examples of the questions to ask, and how to draw the answers
- Standard pictures for circles, rectangles, lengths and sets
- Connections to addition, multiplication and division concepts

Show 3/4 of each shape or group

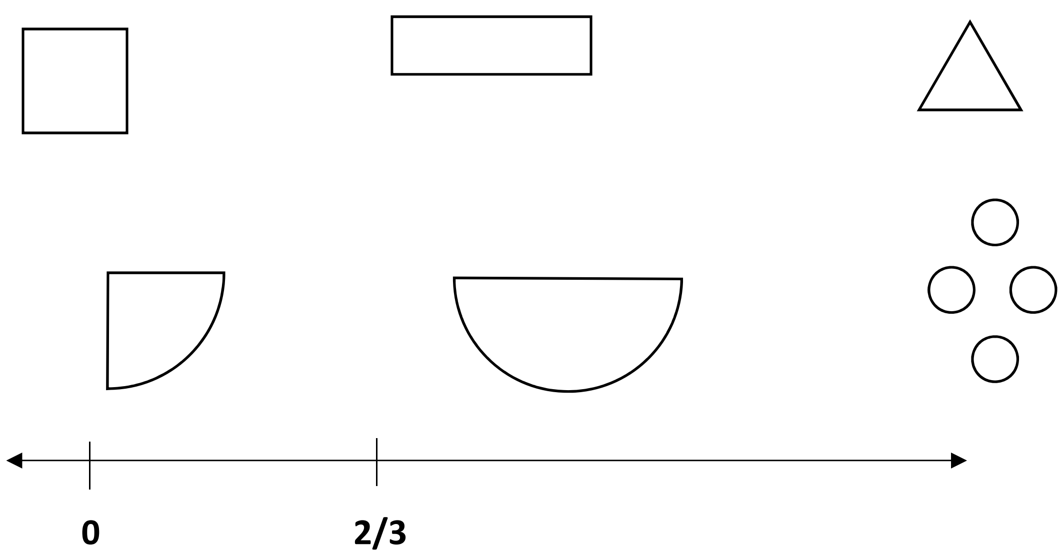


Show 3/4 in 3 different ways on grid paper.

If this shape shows 1/3, what shape shows the whole?



If this shape shows 2/3, what shape shows the whole?



Concept 2—the Standards way of looking at a fraction:

> Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

Make improper fractions using the process implied in this explanation. Ask:

How can you find [1/4] of a whole? OR How do you know this is [1/4] of a whole?
How can you use that [1/4] to show [7/4] of a whole?

> Do this with several fractions where you start with the version with 1 in the numerator (sometimes called a unit fraction) and then change the numerator to make a new fraction. Do this with several units, especially the standard ones: circle, square and/or rectangle, strip of paper of a given length, a group of counters.

Things you might want to take notes on:

- Examples with circles, squares, rectangles, lengths and counters
- Ways to connect circles to counters
- Representations with numbers
- Standards-type questions

Show 7/4 of:

A unit circle

A unit square

12 counters

On a number line:

Concept 3: Fractions can have different names—some fractions are **equivalent**. Fractions are **equal** (**equivalent**, **the same**—children should learn that all of these are different ways of saying the same thing) if they show the *same amount* when made from the *same whole*. Try out and talk about:

Make equivalent fractions for [2/3] using <u>a circle</u> to show 1 whole unit.
Make equivalent fractions for [2/3] using <u>a paper square</u> to show 1 whole unit.
Make equivalent fractions for [2/3] using <u>grid paper</u> to show 1 whole unit.
Make equivalent fractions for [2/3] using <u>a paper strip or number line</u> to show 1 whole unit.
Make equivalent fractions for [2/3] using a <u>group of counters</u> to show 1 whole unit.

> Change the [fraction] and the <u>whole</u> to make a lot of questions. Emphasize ways of making equivalent fractions that involve folding or splitting the parts of a simplified fraction into smaller fractional pieces.

Things you might want to take notes on:

- The 2-dimensional strategy for making equivalent fractions for paper squares
- The additional planning needed to make equivalent fractions using grid paper and counters (that is not needed for paper squares, strips and fraction circles)
- The multiplication explanation and connections

Concept 3, continued: After children have learned how to make equivalent fractions from simplified fractions, ask questions where children find the simplified fraction for a complex fraction. For example:

Use the <u>fraction circle material</u> to find a fraction that is equivalent to [3/6] and has a smaller number of parts.
Use a <u>paper square</u> to find a fraction that is equivalent to [3/6] and has a smaller number of parts.
Use a <u>grid</u> to find a fraction that is equivalent to [3/6] and has a smaller number of parts.
Use a <u>number line</u> to find a fraction that is equivalent to [3/6] and has a smaller number of parts.
Use <u>counters</u> to find a fraction that is equivalent to [3/6] and has a smaller number of parts.

Vary the questions by changing the <u>material representation</u> and the [fraction]

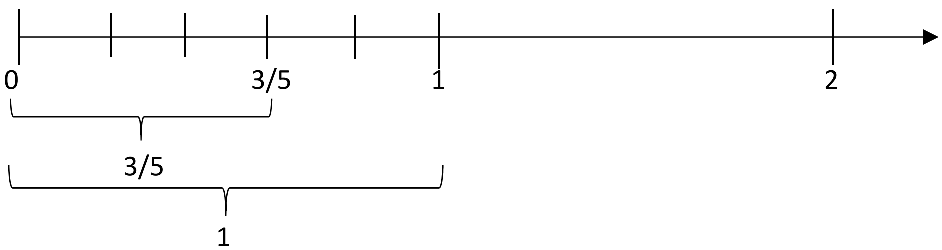
Things you might want to take notes on:

- Strategies for finding simplified fractions
- The grouping division explanation for simplifying fractions

Concept 4: A fraction tells an amount; it is one number, not two. We write a fraction using two whole numbers, but the fraction itself has one value.

Ways of knowing:

- In a fraction, the denominator tells the size of the parts, and the numerator tells how many parts. This is similar to how place value works: the 3 in 300 means 3 parts of size 1 hundred: 2 hundred(s); In the fraction 3/5, the 3 means 3 parts of size 1 fifth. Writing it out as **3 fifths** emphasizes the roles of the 3 and the /5 in the fraction.
- Showing fractions on a number line emphasizes their value in comparison to whole numbers:



A useful thing to remember about number lines is that each number is the name both of a point on the number line and of the distance from 0 to that point.

Adding fractions with unlike denominators:

•The fractions must be shown as **fractions of the same whole** in order for adding or subtracting to make sense.

•We can estimate the sum without finding the numerical answer, because **a fraction tells an amount.** Estimating help us to remember that each fraction tells us one number, not two.

•To get a numerical answer, we need to get **equivalent fractions** with the same denominator

The process of showing equivalent fractions with a square or rectangular whole reinforces the multiplication process for finding equivalent fractions with the same denominator.

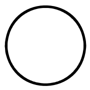
Multiplying fractions

3 × 4 means 3 taken 4 times:

a group has 3 wholes in it, and I take 4 groups

$\frac{1}{2} \times \frac{1}{3}$ *must mean* $\frac{1}{2}$ taken $\frac{1}{3}$ times:

a group has $\frac{1}{2}$ **of a whole** unit in it, and I take $\frac{1}{3}$ **of a group**

 is $\frac{1}{2}$ of . $\frac{1}{2} \times \frac{1}{3}$ is  which is $\frac{1}{3}$ of .

$\frac{4}{5} \times \frac{2}{3}$ means a group has $\frac{4}{5}$ of a whole unit in it. Take $\frac{2}{3}$ of a group.

- When we multiply two numbers, one tells how much of a whole is in a group, and the other tells how many groups. The fractions are fractions **of two different wholes**. One is a fraction of a unit whole (the answer is also a fraction of a unit whole), and the other is a fraction of the other fraction.
- Making diagrams for fraction multiplication is easiest with a square or rectangle as the whole (least messy)

Dividing fractions:

12÷3 *can mean two things*

Usually we say: 12 things shared among 3 groups (people) or

12 things are in 3 groups, how much is in each single group?

Sometimes we say: How many 3's are in 12? or

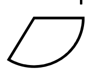
How many groups of 3 can be made from 12?

We can figure out $\frac{1}{2} \div \frac{1}{3}$ by saying: $\frac{1}{2}$ of a whole unit is $\frac{1}{3}$ of a whole group. How much of a whole unit is in in a whole group? (called partitive division)

 is 1/3 of what whole group? How many whole units is that group?

Here, the first fraction: the dividend (1/2) is a fraction of a whole unit. The second fraction: the divisor (1/3) is a fraction of a whole group (different whole). The quotient (answer) is a number (fraction) of a whole unit again (same whole as dividend).

We can also figure out $\frac{1}{2} \div \frac{1}{3}$ by saying: How many groups that are each $\frac{1}{3}$ of a whole unit can be made from $\frac{1}{2}$ of a whole unit? (called measurement or quotative division)

How many  can you make from  ?

Here, the first fraction: the dividend (1/2) is a fraction of a whole unit. The second fraction: the divisor (1/3) is also a fraction of a whole unit (same whole). The quotient (answer) is a number (fraction) of a whole group (different whole).

Most of the harder word problems you'll find look something like this:

Maddy had a piece of ribbon that was $3\frac{1}{2}$ yards long. She used this ribbon to make bows. Each bow was made from a piece of ribbon that was $\frac{3}{4}$ yard long. How many bows could she make?

Which kind of division is that? (measurement)