**A SSS proof (the one that isn't Euclid's)**

Let $\Delta ABC$ and $\Delta DEF$ be triangles such that

(1) $\overline{AB} \cong \overline{DE}$

(2) $\overline{BC} \cong \overline{EF}$

(3) $\overline{AC} \cong \overline{DF}$

By Axiom 4, construct a triangle $\Delta A'EF$ such that

(4) $A'$ lies on the opposite side of $\overleftrightarrow{EF}$ from *D*.

(5) $\angle ABC \cong \angle A'EF$

(6) $\overline{AB} \cong \overline{A'E}$

and hence,

(7) $\Delta ABC \cong \Delta A'EF$

Since the triangles are congruent, we also know that:

(8) $\overline{AC} \cong \overline{A'F}$

which, together with line 3: $\overline{AC} \cong \overline{DF}$ implies that

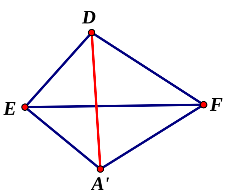
(9) $\overline{A'F} \cong \overline{DF}$ (common notion, transitivity)

Likewise, transitivity together with lines 6 and 1: $\overline{AB} \cong \overline{A'E}$, $\overline{AB} \cong \overline{DE}$ give us that

(10) $\overline{A'E} \cong \overline{DE}$

We now consider 3 cases:

case 1: $\overline{A'D}$ intersects $\overline{EF}$ at a point other than an endpoint




Since $\overline{A'E} \cong \overline{DE}$ (10), the triangle $\Delta EDA'$ is isosceles, and by theorem 3.5, we can conclude that

(11) $\angle EDA' \cong \angle EA'D$

Similarly, since $\overline{A'F} \cong \overline{DF}$ (9), the triangle $\Delta FDA'$ is isosceles, and

(12) $\angle FDA' \cong \angle FA'D$ (Thm 3.5)

Now, in case 1,

$\angle EDA' + \angle FDA' = \angle EDF$ and

$\angle EA'D + \angle FA'D = \angle EA'F$

Using the congruences in (11) and (12), and the common notion that adding equal amounts gives equal results, we can conclude that

(13) $\angle EDF \cong \angle EA'F$

Hence, by SAS,

(14) $\Delta DEF \cong \Delta A'EF$

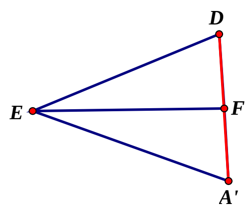
Where the side, angle and side are those given in lines 10, 13 and 9.

Using transitivity together with lines 14 and 7: $\Delta DEF \cong \Delta A'EF$ and $\Delta ABC \cong \Delta A'EF$, we conclude

(15) $\Delta ABC \cong \Delta DEF$

case 2: $\overline{A'D}$ intersects $\overline{EF}$ at a point other than an endpoint.

Without loss of generality, we may assume that $\overline{A'D}$ includes point *F*.




Since $\overline{A'E} \cong \overline{DE}$ (10), the triangle $\Delta EDA'$ is isosceles, and by theorem 3.5, we can conclude that

(16) $\angle EDA' \cong \angle EA'D$

Since *F* lies on $\overline{A'D}$, we can rewrite line (15) to say:

(17) $\angle EDF \cong \angle EA'F$

Hence, by SAS,

(18) $\Delta DEF \cong \Delta A'EF$

Where the side, angle and side are those given in lines 10, 17 and 9:

S: $\overline{A'E} \cong \overline{DE}$

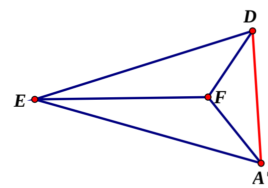
A: $\angle EDF \cong \angle EA'F$

S: $\overline{A'F} \cong \overline{DF}$

Using transitivity together with lines 18 and 7: $\Delta DEF \cong \Delta A'EF$ and $\Delta ABC \cong \Delta A'EF$, we conclude that

(19) $\Delta ABC \cong \Delta DEF$

case 3: $\overline{A'D}$ intersects $\overleftrightarrow{EF}$ but does not intersect $\overline{EF}$. Without loss in generality, we may assume that $\overline{A'D}$ intersects $\overrightarrow{EF}$




As in case 1, since $\overline{A'E} \cong \overline{DE}$ and $\overline{A'F} \cong \overline{DF}$, the triangles $\Delta EDA'$ and $\Delta FDA'$ are isosceles, and we can use thm 3.5 to conclude that

(20) $\angle EDA' \cong \angle EA'D$

(21) $\angle FDA' \cong \angle FA'D$

In case 3:

$\angle EDA' - \angle FDA' = \angle EDF$ and

$\angle EA'D + \angle FA'D = \angle EA'F$

Using the congruences in (20) and (21), and the common notion of subtracting equals, we can conclude that

(22) $\angle EDF \cong \angle EA'F$

Hence, by SAS,

(23) $\Delta DEF \cong \Delta A'EF$

Where the side, angle and side are those given in lines 10, 22 and 9.

By (7), (23) and transitivity

(24) $\Delta ABC \cong \Delta DEF$

Thus in all cases, we conclude $\Delta ABC \cong \Delta DEF$ QED.