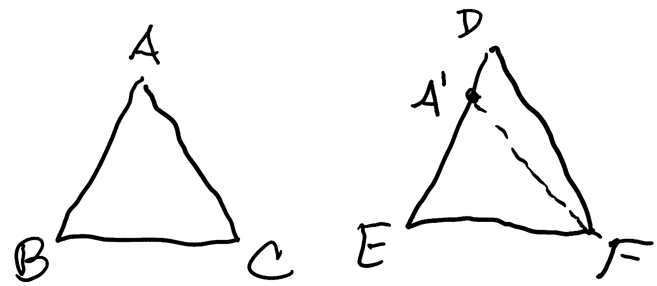
ASA



Let ΔABC and ΔDEF be triangles such that

$\angle ABC \cong \angle DEF$

$\overline{BC} \cong \overline{EF}$

$\angle BCA \cong \angle EFD$ (1)

By Axiom 2 (or 4), there is a point A' on $\overrightarrow{ED}$ such that $\overline{AB} \cong \overline{A'E}$

By SAS $\Delta ABC \cong \Delta A'EF$

S: $\overline{AB} \cong \overline{A'E}$

A: $\angle ABC \cong \angle DEF = \angle A'EF$

S: $\overline{BC} \cong \overline{EF}$

So $\angle ACB \cong \angle A'FE$ (2)

Thus, using transitivity with lines (1) and (2) we get $\angle EFA' \cong \angle EFD$

Since angles $\angle EFA'$ and $\angle EFD$ share a side, are congruent, and are measured in the same direction, the other side is also shared: $\overrightarrow{FA'} = \overrightarrow{FD}$

We know that both A' and D lie on $\overrightarrow{ED}$ and both A' and D lie on $\overrightarrow{FA'} = \overrightarrow{FD}$

By theorem 1 (lines intersect in at most one point), A' and D must be the same point.

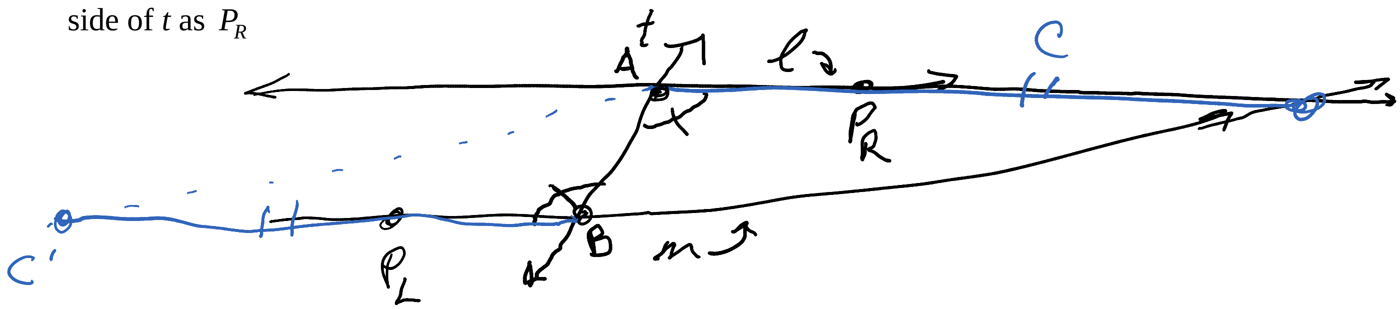
Hence $\Delta ABC \cong \Delta DEF$ QED

Parallel lines exist

Let l and m be lines, and let t be a line that intersects l and m in points A and B respectively such that $\angle ABP_L \cong \angle BAP_R$ where P_L and P_R lie on m and l respectively, and P_L and P_R lie on opposite sides of t.

Either l and m intersect or they are parallel.

Suppose (case 1) that l and m intersect.
Without loss in generality, we may assume that their point of intersection C lies on the same side of t as P_R



Now, by theorem 1, there exists a point C' on m such that $\overline{AC} \cong \overline{BC'}$
By SAS $\Delta BAC \cong \Delta ABC'$

S: $\overline{AB} \cong \overline{BA}$

A: $\angle ABP_L \cong \angle BAP_R$

S: $\overline{AC} \cong \overline{BC'}$

Hence $\angle ABC \cong \angle BAC'$
Thus $\angle BAC' + \angle BAC \cong \angle ABC + ABC' = 180°$ and so C', A, and C must all lie on a straight line (by the straight line axiom?) and the line through A and C is l, so $C' \in l$
So we have that $C \in l \cap m$ and $C' \in l \cap m$, but C and C' can't be the same point because they lie on opposite sides of t, so this contradicts theorem 1.

Thus we must conclude that l and m do not intersect, and hence that they are parallel.