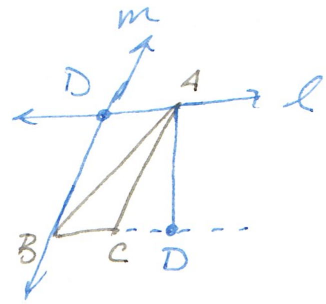
Area of a triangle formula proof:



(1) Let $\triangle ABC$ be a triangle with base $\overline{BC}$

(2) Let l be the line through l, parallel to $\overline{BC}$
(parallel lines exist)

(3) Let m be the line through A, parallel to $\overline{AC}$
(parallel lines exist)

Key ideas
start from $\triangle$
and make $\square$
with same
base & height

(4) Let D be the intersection of l and m.

(5) Then $\square$ ACBD is a parallelogram (2 & 3, defn of parallelogram)

(6) Construct $\overline{AD} \perp \overleftrightarrow{BC}$ such that D lies on $\overline{BC}$
($\perp$ lines exist)

(7) $\overline{AD}$ is the height of $\triangle ABC$ relative to base $\overline{BC}$ (defn)

(8) $\overline{BC}$ is a base of $\square$ ACBD (because it is a side)

(9) $\overline{AD}$ is a height of $\square$ACBD relative to base $\overline{BC}$ ($\perp$ to $\overline{BC}$,
endpoints on $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$)

(10) area $\square$ ABCD $= m(\overline{BC}) \cdot m(\overline{AD})$

use $\square$ area thm

(11) $\triangle ACB \cong \triangle BDA$ (parallelogram $\triangle$ pair thm)

← area axiom

(12) area($\triangle$ACB) = area($\triangle$BDA)

because the $\triangle$s
are congruent

(13) area $\square$ABCD = area($\triangle$ACB) + area($\triangle$BDA)
= area($\triangle$ACB) + area($\triangle$ACB)
= 2 area($\triangle$ACB)

area $\triangle = \frac{1}{2}$ area $\square$

(14) $\frac{1}{2}$ area $\square$ ABCD = area $\triangle$ ACB)

(15) $\frac{1}{2}\, m(\overline{BC})\, m(AD) = \frac{1}{2}\, b \cdot h =$ area $\triangle ACB$