The trickiest case is proving that the segment  is contained in  if  and .

This is tricky because we don’t have any theorems that tell us about what happens if one end of a segment is on a line and the other end isn’t.

Hint 0 (in class): So, often the best thing to try if something seems tricky is a proof by contradiction. Basically, we’re saying “well, if it’s that hard to prove, maybe it’s false. Is there anything that would go wrong if this were false?” So we suppose it’s false, and see what happens. That’s what we discussed in class.

In our in-class discussion, we noted that these are equivalent things to say when supposing it’s false:

Suppose:

 which means the same as  which is basically the same as there is a point

Anyway, in case you hadn’t guessed, that last version is the easiest to work with—largely because we’ve actually named a point with an interesting new property.

So, on to the new hints:

Hint 1: Notice that we now have a point on  that is in  and a point that is in . Look at the definition of separate, and remember that *L* separates  from . You should be able to use this to get a new point that’s in an interesting place. I recommend naming it. I think I’ll call my new point B. I’d be really impressed if you explained how we know that B isn’t any of the named points we have so far.

Hint 2: OK, now you have X, and A, and Y and the new point from hint 1, which I’ll call B. Now, you may notice that X and B are both in some interesting sets. One of the interesting sets is a line segment  which is (I even put it in the definition!) a subset of the line , so *X* and *B* are both on line . What other set are they both in? Interesting huh?

Hint 3: Wow, them both being a in those two sets contradicts something we’ve already proved! (and we’ve only proved one theorem, so it should be pretty easy to figure out what theorem that is). Check it out, a contradiction means that the “suppose” statement we made back in hint 0, so that “suppose” statement is false. If that suppose statement is false, what has to be true?

Congratulations—you should be done with proving the tricky case now!