Geometry

Axiom. Given a pair of points, A, B, there is a non-negative real number called the distance between the points, which may be denoted $d(A,B)$. The length of a line segment, $m(\overline{AB})$ is defined to be the distance between the endpoints. The distance function satisfies the following conditions:

- $d(A,B)=0$ if and only if $A=B$
- For any point C, $d(A,C)+d(C,B)\geq d(A,B)$
- $d(A,C)+d(C,B)=d(A,B)$ of and only if C lies on $\overline{AB}$

Line segments are congruent if and only if their lengths are equal.

Two Rays Separate (Theorem): Given rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ with a common starting point, the rays separate the plane into two sides such that any segment connecting a point from one side to the other must intersect the union of the rays. The sides are: $\overleftrightarrow{AB}_C \cap \overleftrightarrow{AC}_B$ (where $\overleftrightarrow{AB}_C$ is the side of $\overleftrightarrow{AB}$ containing C, and $\overleftrightarrow{AC}_B$ is the side of $\overleftrightarrow{AC}$ containing B), and $\overleftrightarrow{AB}_{\overline{C}} \cup \overleftrightarrow{AC}_{\overline{B}}$ (where $\overleftrightarrow{AB}_{\overline{C}}$ is the side of $\overleftrightarrow{AB}$ that does not contain C, and $\overleftrightarrow{AC}_{\overline{B}}$ is the side of $\overleftrightarrow{AC}$ that does not contain B).

Defn. An **angle** consists of a pair of rays, with common starting point rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ together with one of the sides of the union of the rays. The **interior** of the angle is the side of the plane specified by the angle. If the side specified is $\overleftrightarrow{AB}_C \cap \overleftrightarrow{AC}_B$, then we call the angle a traditional angle, and we denote it $\angle BAC$, if the side specified is $\overleftrightarrow{AB}_{\overline{C}} \cup \overleftrightarrow{AC}_{\overline{B}}$, then we call the angle a reflex angle, and we denote it $r\angle BAC$. In cases where we wish to denote an angle without specifying whether it is a traditional or a reflex angle, we will denote it $\measuredangle BAC$. Note that this last notation is more commonly used to denote angle measure.

Axiom. Every angle can be measured, and the measure of an angle is a number between 0º and 360º. If the rays of the angle are equal, then the angle measure is 0° (for a traditional angle) or 360° (for a reflex angle), and if the rays form a straight line then the angle measure is 180°. Angles have equal measure if and only if the angles are congruent. If two angles $\measuredangle BAC$ and $\measuredangle CAD$ share a ray, and if the union of the angles (including their interiors) is equal to $\measuredangle BAD$, then $m\measuredangle BAC + m\measuredangle CAD = m\measuredangle BAD$, where m denotes the measure of the angle.

Side Angle Side Congruence: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Angle Side Angle Congruence: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Side Side Side Congruence: If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

Hypotenuse Leg Congruence: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

Defn. A triangle is **iscosceles** if two sides are congruent.

Isosceles Triangle Angles: A triangle is isosceles if and only of two angles are congruent.

Triangle Side-Angle Correspondence: In a triangle, with two angles unequal, the side opposite the larger angle is larger than the side opposite the smaller angle.

Defn. Two lines are perpendicular if there is an angle, whose rays are subsets of the lines, that measures 90°.

Perpendicular lines (proof and construction): Given a point A and line l, there is a unique line m that contains A and is perpendicular to l.

Chord-Radius Perpendicular: For ever chord in a circle, there is a radius of the circle that is perpendicular to the chord.

Point to line distance (theorem and defn): Given a point A and line l, the shortest distance from A to a point on l is the distance to the point of intersection of l with the line perpendicular to l that passes through A. This shortest distance is called the **distance between A and l.**

Triangle Angle Sum: The sum of the measures of the angles in a triangle is 180°.

Theorem: If a line passing through a point of a circle is perpendicular to the radius to that point, then the line is tangent to the circle.

Theorem: A radius is perpendicular to a chord (that is not a diameter) if and only if it bisects the chord.

Theorem: Two chords of a circle have equal length if and only if they are at equal distances from the center.

Theorem (Lemma): The measure of an angle inscribed in a circle with one side a diameter of the circle is half of the measure of the central angle that subtends the same arc of the circle.