

Defn. A **circle** is a set of points in the plane equidistant from a fixed point called the **center**.

Defn. A **radius** of a circle is a line segment that joins the center of the circle and a point on the circle.

Defn. A **chord** is a line segment that joins two points of the circle.

Defn. A **diameter** of a circle is a chord that passes through the center of the circle

Defn. A line is **tangent** to a circle if it intersects the circle at exactly one point. The point of intersection\* is called the **point of tangency**.

\*The definition of intersection is inherited from the set theory definition of intersection. Circles and lines are both sets of points, and hence their intersection is the set of points shared by both sets.

Axiom. A line divides the plane into two non-intersecting sets called **sides** (that also do not intersect the line) with the properties that:

- if any two points are in the same side of the line, then the segment that joins them lies entirely in that side of the line, and
- if any two points are on opposite sides of the line, then the segment that joins them intersects the line.

Defn. Given a circle  $C$ , and a line  $l$  with sides  $S$  and  $T$  that intersects the circle at two points  $A$  and  $B$ , then the intersections  $(l \cup S) \cap C$  and  $(l \cup T) \cap C$  are **arcs** of the circle.

Defn. The arcs determined by a diameter of a circle are **semicircles**.

Defn. An angle between two radii of a circle is a **central angle** of the circle.

Defn. If  $A$ ,  $B$ , and  $C$  are points on a circle, then the angle  $\angle ABC$  is an **inscribed angle** of the circle.

Thm. A tangent line to a circle is perpendicular to the radius to the point of tangency.

Thm. In a circle, a radius bisects a chord that is not a diameter if and only if the radius and the chord are perpendicular to each other.

Thm. If two circles are externally tangent, then the line segment joining their centers contains the point of tangency.

Thm. The circumference of a circle is  $2\pi r$ , where  $r$  is the length of a radius.

Thm. The area of a circle is  $\pi r^2$ , where  $r$  is the length of a radius.

Thm. Through every point on a circle, there is a tangent line to the circle.

Thm. The measure of a central angle of a circle is double that of the inscribed angle that subtends the same arc.