Geometry Assignment (due Friday)

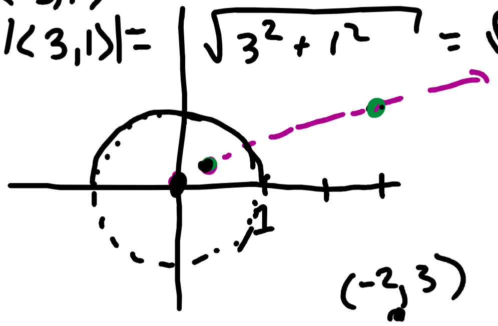
1. Calculate the reflection of each of these points when inverted in the unit circle $x^2 + y^2 = 1$

a. (3,1) $\to \langle 3,1 \rangle$

$|\langle 3,1 \rangle| = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\dfrac{\langle 3,1 \rangle}{\sqrt{10}^2} = \langle .3, .1 \rangle$

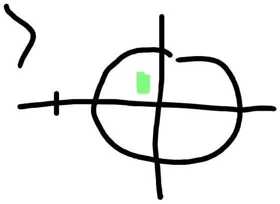
$\to (.3, .1)$



b. (-2,3)

$|\langle -2,3 \rangle| = \sqrt{4+9}$

$\dfrac{\langle -2,3 \rangle}{|\langle -2,3 \rangle|^2} = \left\langle -\dfrac{2}{13}, \dfrac{3}{13} \right\rangle$



$\left(-\dfrac{2}{13}, \dfrac{3}{13} \right)$

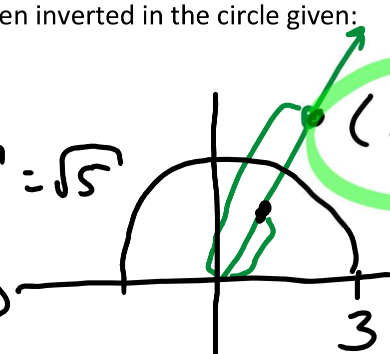
2. Calculate the reflection of each of these points when inverted in the circle given:

a. (1,2) in the circle $x^2 + y^2 = 9$

$\langle 1,2 \rangle$

$|\langle 1,2 \rangle| = \sqrt{1+4} = \sqrt{5}$

$\langle 1,2 \rangle \cdot \dfrac{3^2}{\sqrt{5}^2} = \left\langle \dfrac{9}{5}, \dfrac{18}{5} \right\rangle$



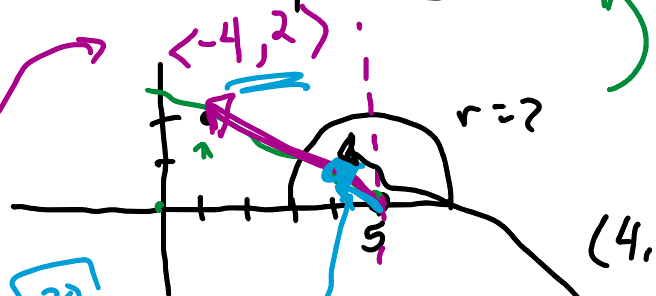
$\left(\dfrac{9}{5}, \dfrac{18}{5} \right)$

b. (1, 2) in the circle $(x-5)^2 + y^2 = 4 = r^2$

$x - h$

$5 = h$

$\langle 1,2 \rangle - \langle 5,0 \rangle = \langle -4, 2 \rangle$



$|\langle -4,2 \rangle| = \sqrt{16^2 + 2^2} = \sqrt{20}$

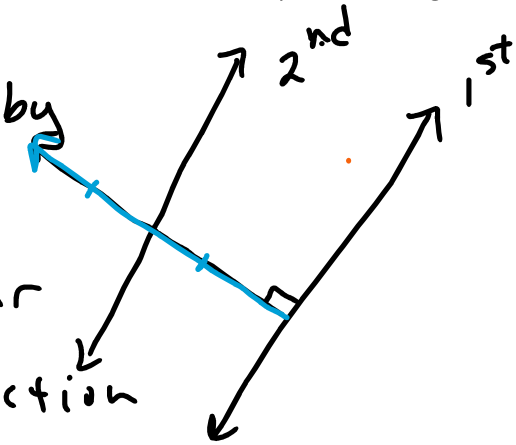
$\langle -4,2 \rangle \cdot \dfrac{2^2}{\sqrt{20}^2} = \langle -4,2 \rangle \dfrac{4}{20}$

$= \langle -4,2 \rangle \dfrac{1}{5} = \left\langle -\dfrac{4}{5}, \dfrac{2}{5} \right\rangle$

$\langle 5,0 \rangle + \left\langle \overset{-.8}{-\dfrac{4}{5}}, \overset{.4}{\dfrac{2}{5}} \right\rangle = \langle 4.2, .4 \rangle$

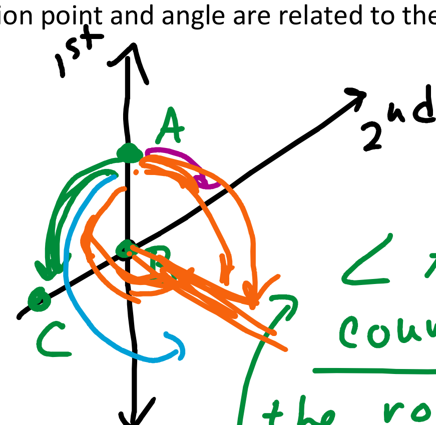
$(4.2, .4)$

3. Explain what transformation you get by doing two reflections across parallel reflection lines. If the transformation is a translation, describe how the translation vector is related to the parallel lines. If the transformation is a rotation, describe how the rotation point and angle are related to the reflection lines.



translation by a vector perpendicular to the reflection lines, and twice as long as the distance between the lines. (from first refl. line towards 2nd)

4. Explain what transformation you get by doing two reflections across intersection reflection lines. If the transformation is a translation, describe how the translation vector is related to the parallel lines. If the transformation is a rotation, describe how the rotation point and angle are related to the reflection lines.

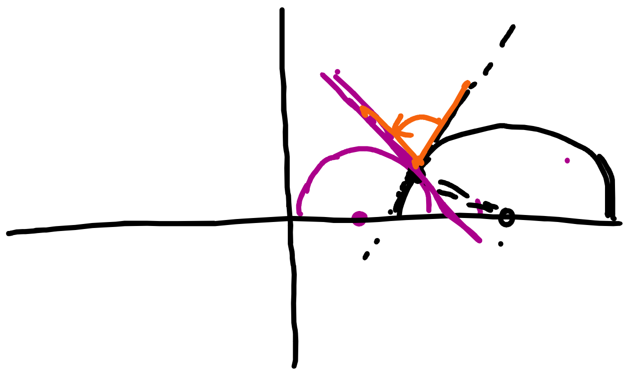


A rotation around the intersection of the two reflection lines. by an angle that is 2x the angle between the reflection lines.

If you pick a point, A, on the first reflection line (call the intersection point B), and C ∈ second line such that ∠ABC is counterclockwise

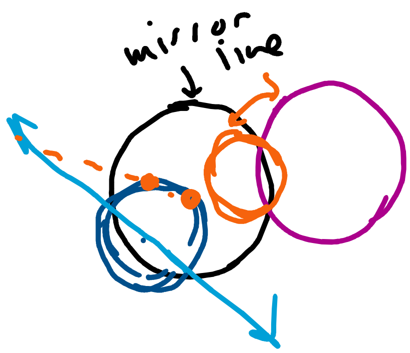
the rotation 2x m∠ABC counterclockwise

5. Describe how we measure angles between intersecting circles (for instance, hyperbolic lines that are half-circle shaped).

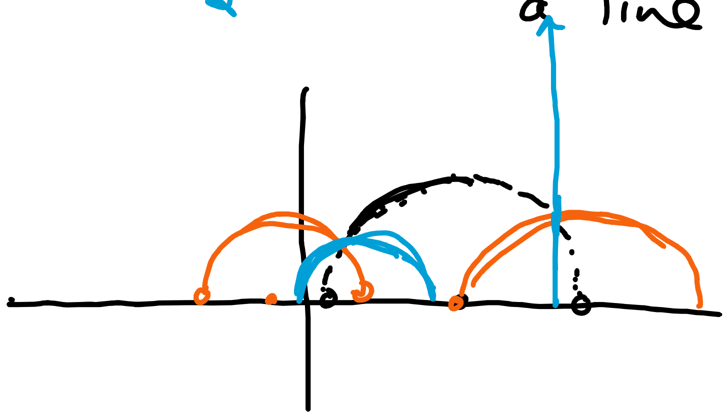


measure the angle between the tangent lines to the circles at the point of intersection

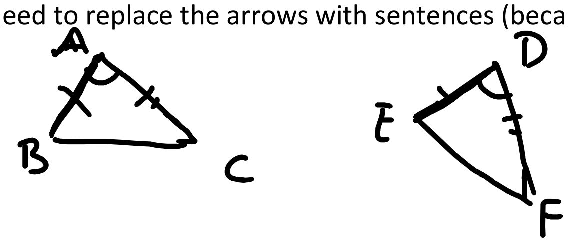
6. If you invert/reflect a circle across another circle, what shape(s) is the reflection?



A circle unless the circle goes through the center of the inversion circle, and then you get a line



7. Explain how the SAS theorem can be proved using the Application axiom. You can use the in class outline from Oct 11 (Look in Zoom in Canvas). You need to replace the arrows with sentences (because ____ and ___, therefore ____).



Given $\triangle ABC$, $\triangle DEF$

such that $\overline{AB} \cong \overline{DE}$

and $\overline{AC} \cong DE$

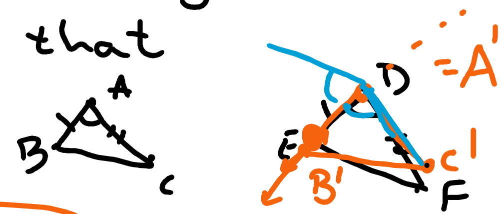
and $\angle BAC \cong EDF$

By Application There is a congruence transformation such that

A maps to D

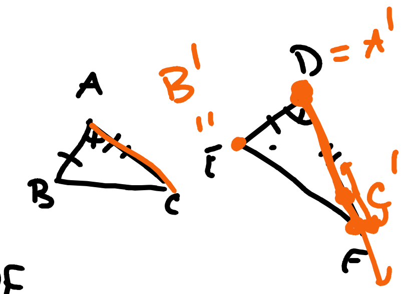
B maps to $B' \in \overrightarrow{DE}$

C maps to C' that is on the same side of $\overleftrightarrow{DE}$ as F



Because $\overline{AB} \cong \overline{DE}$, $A' = D$, $B' \in \overrightarrow{DE}$

So $B' = E$ ←



Because $\overline{A'B'} = \overline{DE}$ and $\angle ABC \cong \angle EDF$ ($\angle A'B'C' \cong \angle EDF$)

and C' and F are on the same side of $\overleftrightarrow{DE}$, so $C' \in \overrightarrow{DF}$

Because $\overline{AC} \cong \overline{DF}$ ($\overline{A'C'} \cong \overline{DF}$)

an $C' \in \overrightarrow{DF}$ (and $A' = D$)

so $C' = F$

So my congruence transfomatin maps

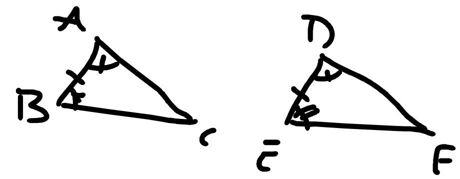
$A \to D$

$B \to E$. So $\triangle ABC \cong \triangle DEF$

$C \to F$

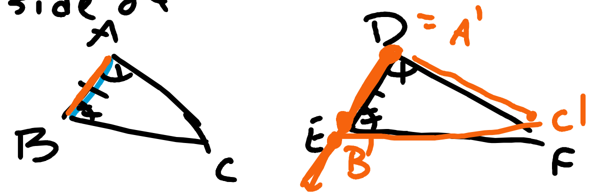
Why Application?

8. Explain the ASA theorem can be proved using the Application axiom.



Given $\triangle ABC$ and $\triangle DEF$
such that $\angle BAC \cong \angle EDF$
$\overline{AB} \cong \overline{DE}$
$\angle ABC \cong \angle DEF$

By Application, there is a congruence transformation that maps
A to $A' = D$
B to $B' \in \overrightarrow{DE}$
C to C' on the same side of $\overleftrightarrow{DE}$ as F



Because $A' = D$, $\overline{AB} \cong \overline{DE}$ ($\overline{A'B'} \cong DE$)
and $B' \in \overrightarrow{DE}$,
so $B' = E$ ←

Because $\angle BAC \cong \angle EDF$ ($\angle B'A'C' \cong \angle EDF$)
and $B' = E$, $A' = D$ and
C' and F are on the same side of $\overleftrightarrow{DE}$
so $C' \in \overrightarrow{DF}$ ($\angle EDF = \angle EDC'$)

Because $\angle ABC \cong \angle DEF$ ($\angle A'B'C' \cong \angle DEF$)
and $B' = E$, $A' = D$ and
C' and F are on the same side of $\overleftrightarrow{DF}$
so $C' \in \overrightarrow{EF}$ ($DEF = \angle DEC'$)

$\triangle A'B'C' = \triangle DEF$

so $C' \in \overrightarrow{DF} \cap \overrightarrow{EF}$, so $C' = F$ ←. Therefore $\triangle ABC \cong \triangle DEF$