An updated version of Euclid’s Postulates

Postulates about straight lines:

Postulate 1: To draw a straight line from any point to any point.

Postulate 2: To produce a finite straight line continuously in a straight line

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Axiom 1 (line uniqueness): Through any two distinct points there is exactly one line.

Common Notations (these apply to segments, angles and shapes)

CN 1: Things which are equal to the same thing are also equal to each other

CN 2: If equals be added to equals the wholes are equal

CN 3: If equals be subtracted from equals the remainders are equal

CN 4: Things that coincide with one another are equal to one another

CN 5: The whole is greater than the part.

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Axiom 2 (Measurements): Segments, angles and areas can be measured, and their measurements correspond to non-negative real numbers.

If a segment is composed of two segments that intersect only at a common endpoint, or an angle is composed of two angles that intersect only in a common side, or a shape is composed of two shapes that intersect only in a common boundary curve, then the measurement of the object is equal to the sum of the measurements of the component parts.

Angles and segments have the same measurement in both directions: $m(\overline{AB}) = m(\overline{BA})$ and $m\angle ABC = m\angle BAC$

Postulates for comparing two different things:

Postulate 3: To describe a circle with any center and distance.

Postulate 4: That all right angles are equal to one another

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Axiom 3 (Application/Congruence by transformations): There exist a set of congruence transformations (functions) that map points in the plane to points in the plane in such a way that lines map to lines, and measurements of segments, angles and areas are preserved.

Axiom 4 (Separation): Infinite lines, triangles and circles separate the plane into two portions or sides such that any line or arc of a circle that joins a point on one side to a point on the other side intersects the separating figure.

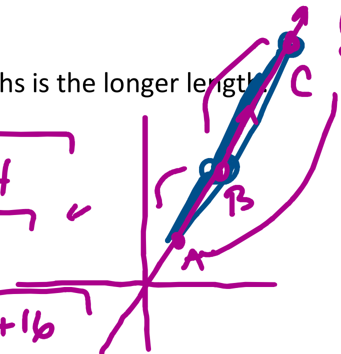
Testing two models to see if they satisfy the measurement axiom:

1. Test the hypothesis lines in the x-y plane with analytic geometry, with the usual distance formula $d((x_1,y_1),(x_2,y_2)) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ satisfy Axiom 2 by doing the following:

- Write an example of an equation of a line in $y = mx + b$. Choose random numbers for *m* and *b* by rolling a die.
- Find 3 distinct points on the line *A, B¸* and *C*.
- Calculate all of the distances *AB*, *BC*, and *AC*.
- Calculate whether the sum of the two smaller lengths is the longer length.

$y = 2x + 4$

	x	y
$A=$	$(1,$	$6)$
$B=$	$(2,$	$8)$
$C=$	$(4,$	$12)$

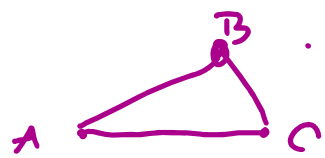


$AB = \sqrt{(2-1)^2 + (8-6)^2} = \sqrt{1+4} = \sqrt{5}$

$BC = \sqrt{(2-4)^2 + (8-12)^2} = \sqrt{4+16} = \sqrt{20}$

$AC = \sqrt{(1-4)^2 + (6-12)^2} = \sqrt{9+36} = \sqrt{45}$

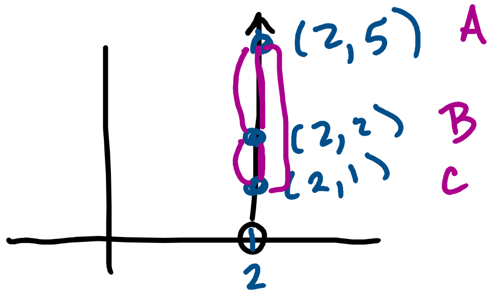
$\sqrt{5} + \sqrt{20} = \sqrt{5} + 2\sqrt{5} = 3\sqrt{5} = \sqrt{9 \cdot 5} = \sqrt{45}$



2. Test the hypothesis that lines in upper half plane hyperbolic geometry with the distance formulas given on page 18 of the textbook by:

a. The hyperbolic distance between two points that lie on a vertical line $x = a$ is given by $\boxed{d((a, y_1),(a, y_2)) = \left|\ln \frac{y_1}{y_2}\right|}$

- Write an example of an equation of an upper half plane U-Line of the form $x = a$. Choose a random number for *a* by rolling a die.
- Find 3 distinct points on the line in the upper half plane: *A, B*, and *C*.
- Calculate all of the distances *AB*, *BC*, and *AC*.
- Calculate whether the sum of the two smaller lengths is the longer length.



$AB = \left|\ln \frac{5}{2}\right| = \ln\left(\frac{5}{2}\right)$
$= \ln 5 - \ln 2$

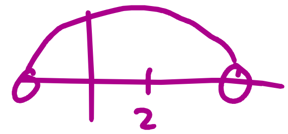
$BC = \left|\ln \frac{2}{1}\right| = \ln\left(\frac{2}{1}\right)$
$= \ln 5 - \ln 1$

$AC = \left|\ln \frac{5}{1}\right| = \ln\left(\frac{5}{1}\right)$
$= \ln 5 - \ln 1$

$AB + BC = (\ln 5 - \ln 2) + (\ln 2 - \ln 1)$
$= \ln 5 - \ln 1 = AC$

b. The hyperbolic distance between two points that line on a circle with center $(c,0)$ and radius r is given by

$$d((a,y_1),(a,y_2)) = \left|\ln\frac{(x_1-c-r)y_2}{(x_2-c-r)y_1}\right|$$



- Write an example of an equation of an upper half plane U-Line of the form $(x-c)^2+y^2=r^2$. Choose random numbers for *c* and *r* by rolling a die.
- Find 3 distinct points on the line in the upper half plane: *A, B*, and *C*.
- Calculate all of the distances *AB*, *BC*, and *AC*.
- Calculate whether the sum of the two smaller lengths is the longer length.

$(x-2)^2+y^2=5^2$

x	y	
0	$\sqrt{21}$	$=A$
1	$\sqrt{24}$	$=B$
5	4	$=C$

$c=2,\ r=5$

$(5-2)^2+y^2=25$

$y^2=16$

$(0-2)^2+y^2=25$

$y^2=21$

$(1-2)^2+y^2=25$

$y^2=24$

$$d((a,y_1),(a,y_2)) = \left|\ln\frac{(x_1-c-r)y_2}{(x_2-c-r)y_1}\right|$$

$(0,\sqrt{21})=A$

$$AB = \left|\ln\frac{(0-2-5)\sqrt{24}}{(1-2-5)\sqrt{21}}\right| = \left|\ln\frac{+7\sqrt{24}}{+6\sqrt{21}}\right|$$

$$= \ln\left(\frac{7\sqrt{24}}{6\sqrt{21}}\right) = \ln 7 + \ln\sqrt{24} - \ln 6 - \ln\sqrt{21} \leftarrow$$

$$BC = \left|\ln\frac{(1-2-5)\,4}{(5-2-5)\sqrt{24}}\right| = \left|\ln\frac{+6\cdot 4}{+2\cdot\sqrt{24}}\right| = \ln\left(\frac{6\cdot 4}{2\sqrt{24}}\right)$$

$$= \ln 6 + \ln 4 - \ln 2 - \ln\sqrt{24} \leftarrow$$

$$AC = \left|\ln\frac{(0-2-5)\cdot 4}{(5-2-5)\sqrt{21}}\right| = \ln\left(\frac{+7\cdot 4}{+2\sqrt{21}}\right)$$

$$= \ln 7 + \ln 4 - \ln 2 - \ln\sqrt{21} \leftarrow$$

$$AB+BC = \ln 7 + \cancel{\ln\sqrt{24}} - \cancel{\ln 6} - \ln\sqrt{21} + \cancel{\ln 6} + \ln 4 - \ln 2 - \cancel{\ln\sqrt{24}}$$

$$= \ln 7 + \ln 4 - \ln\sqrt{21} - \ln 2 = AC$$