An updated version of Euclid's Postulates version 2

Postulates about straight lines:

Postulate 1: To draw a straight line from any point to any point.

Postulate 2: To produce a finite straight line continuously in a straight line

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Axiom 1 (line uniqueness): Through any two distinct points there is exactly one line.

Common Notations (these apply to segments, angles and shapes)

CN 1: Things which are equal to the same thing are also equal to each other

CN 2: If equals be added to equals the wholes are equal

CN 3: If equals be subtracted from equals the remainders are equal

CN 4: Things that coincide with one another are equal to one another

CN 5: The whole is greater than the part.

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Axiom 2 (Measurements): Segments, angles and areas can be measured, and their measurements correspond to non-negative real numbers.

If a segment is composed of two segments that intersect only at a common endpoint, or an angle is composed of two angles that intersect only in a common side, or a shape is composed of two shapes that intersect only in a common boundary curve, then the measurement of the object is equal to the sum of the measurements of the component parts.

Angles and segments have the same measurement in both directions: $m(\overline{AB}) = m(\overline{BA})$ and $m\angle ABC = m\angle BAC$

Postulates for comparing two different things:

Postulate 3: To describe a circle with any center and distance.

Postulate 4: That all right angles are equal to one another

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Axiom 3 (Application/Congruence by transformations): There exist a set of congruence transformations (functions) that map points in the plane to points in the plane in such a way that lines map to lines, and measurements of segments, angles and areas are preserved.

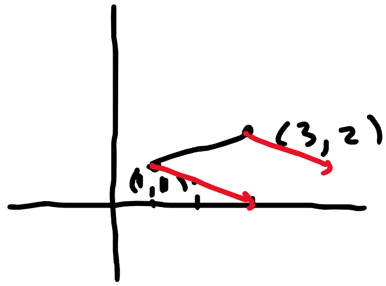
Given any two triangles ΔABC and $\Delta A'B'C'$, there is one of these transformations that maps the point A to A', the point B to a point on $\overrightarrow{A'B'}$, and the point C to a point on the same side of $\overleftrightarrow{A'B'}$ as the point C'

Axiom 4 (Separation): Infinite lines, triangles and circles separate the plane into two portions or sides such that any line or arc of a circle that joins a point on one side to a point on the other side intersects the separating figure.

Testing the analytic plane to see if it satisfies the application axiom:

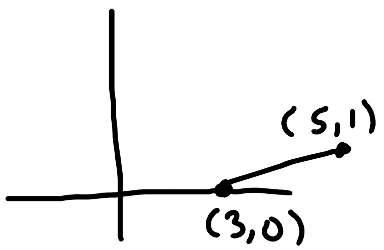
1. Test whether translations, rotations and reflections preserve lengths:

a. Choose a segment that does not lie on either of the coordinate axes, and does not include the origin. Tell the coordinates of the endpoints. Find the length of this segment.



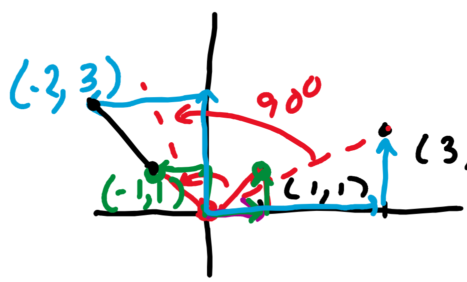
$$\sqrt{(3-1)^2+(2-1)^2}$$
$$=\sqrt{4+1}=\sqrt{5}\ \checkmark$$

b. translate the segment by the vector $\langle 2,-1\rangle$. Tell the coordinates of the new endpoints. Find the length of this segment.

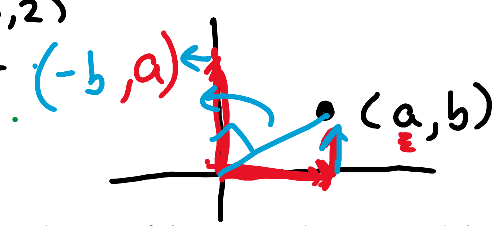


$$\sqrt{(5-3)^2+(1-0)^2}=\sqrt{5}\ \checkmark$$
$$\langle 3,2\rangle+\langle 2,-1\rangle=\langle 5,1\rangle$$
$$\langle 1,1\rangle+\langle 2,-1\rangle=\langle 3,0\rangle$$

c. Rotate the segment by 90° counterclockwise around the origin. Tell the coordinates of the new endpoints. Find the length of this segment.

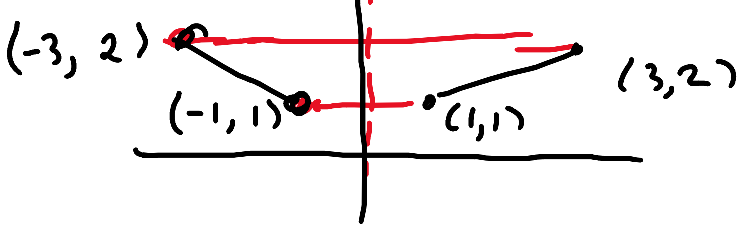


$$\sqrt{(-1--2)^2+(1-3)^2}$$
$$=\sqrt{1+2^2}=\sqrt{5}$$



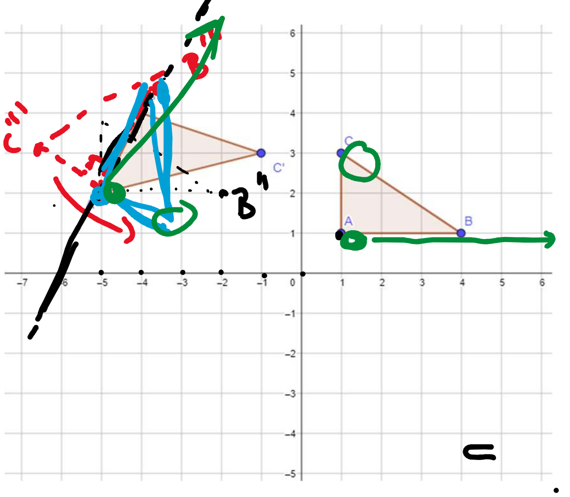
d. Reflect the segment across the y-axis. Tell the coordinates of the new endpoints. Find the length of the segment.

$$\sqrt{(-3--1)^2+(2-1)^2}=\sqrt{5}$$



3. Show that you can use a combination of translations, rotations and reflections to map one triangle so it is close to another. Where "close to" means: Given any two triangles ΔABC and $\Delta A'B'C'$, there is one of these transformations that maps the point A to A', the point B to a point on $\overrightarrow{A'B'}$, and the point C to a point on the same side of $\overleftrightarrow{A'B'}$ as the point C'

a. For the triangles shown below, describe a sequence of translations, rotations and reflections that will map ΔABC close to $\Delta A'B'C'$. Draw out the final image of where ΔABC is mapped to at the end.



Translate $\langle -6, 1 \rangle$

call the new triangle $\Delta A'B''C''$

Rotate around A' by $\angle B''A'B'$

call the new triangle $\Delta A'B'''C'''$

Reflect across $A'B'$

b. Your friend (me) has two triangles ΔABC and $\Delta A'B'C'$. Write instructions for a sequence of translations, rotations and reflections that will map ΔABC close to $\Delta A'B'C'$. You will need to write instructions that work even if you don't know what the vertices of the triangles are.

Translate ΔABC by vector $\overrightarrow{AA'}$

call the new triangle $\Delta A'B''C''$

rotate around A' by angle $\angle B''A'B'$

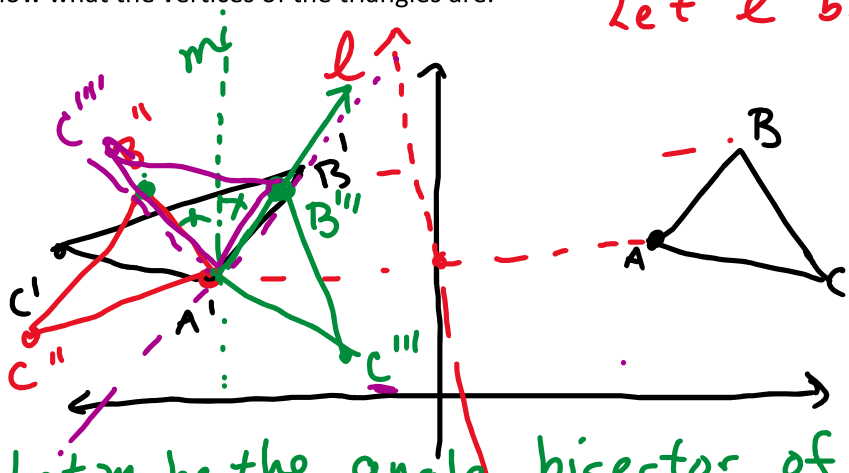
call the new triangle $\Delta A'B'''C'''$

If C''' is on the wrong side of $\overleftrightarrow{A'B'}$

(opposite side from C')

then reflect across $\overleftrightarrow{A'B'}$

c. Your friend (me) has two triangles ΔABC and $\Delta A'B'C'$. Write instructions for a sequence of reflections (only reflections) that will map ΔABC close to $\Delta A'B'C'$. You will need to write instructions that work even if you don't know what the vertices of the triangles are.



Let ℓ be the perpendicular bisector of $\overline{AA'}$

Reflect $\triangle ABC$ across ℓ,

Name the image $\triangle A'B''C''$

Let m be the angle bisector of angle $\angle B'A'B''$. Reflect across m.

Name the image $\triangle A'B'''C'''$

If C''' is not on the same side of $\overleftrightarrow{A'B'}$ as C', then reflect across $\overleftrightarrow{A'B'}$.