Shared Notes, theorems 20, 21, 23

Theorem 20

Given $\overline{AB}$, $\overline{CD}$ such that $\underline{AB} = \underline{CD}$

By Ax 3, there exists an isometry f, such that

$f(A) = C$

$f(B) \in \overrightarrow{CD}$

Because $\underline{AB} = \underline{CD}$

We know that $\underline{AB} = \underline{f(A)f(B)}$ and $\underline{AB} = \underline{Cf(B)}$

By substitution $\underline{CD} = \underline{Cf(B)}$

By defn: $\overrightarrow{CD} = \{\underline{Cf(B)} + \underline{f(B)D} = \underline{CD}$ or $\underline{CD} + \underline{Df(B)} = \underline{Cf(B)}$

Case 1

$\underline{Cf(B)} + \underline{f(B)D} = \underline{CD}$

By Substitution:

$\underline{CD} + \underline{f(B)D} = \underline{CD}$

so $\underline{f(B)D} = 0$

So By Ax 1(c), $f(B) \neq D$

Case 2

$\underline{CD} + \underline{Df(B)} = \underline{Cf(B)}$

By substitution:

$\underline{CD} + \underline{Df(B)} = \underline{CD}$

so $\underline{Df(B)} = 0$

By Ax 1(c), $D = f(B)$

Theorem 21:

Given $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$

By Axiom 3, there exists an isometry f, such that

$f(A) = C$

and $f(B) \in \overrightarrow{CD}$

so by definition of line & ray

$f(B) \in \overleftrightarrow{CD}$

and $C \in \overleftrightarrow{CD}$ so $f(A) \in \overleftrightarrow{CD}$

by thm 9 $\overleftrightarrow{CD} = \overleftrightarrow{f(A)f(B)}$

By thm 18c $\overleftrightarrow{f(A)f(B)} = f(\overleftrightarrow{AB})$

so, $f(\overleftrightarrow{AB}) = \overleftrightarrow{CD}$

means (by def)

$\overleftrightarrow{AB} \cong \overleftrightarrow{CD}$

Thm 23 Given $\overleftrightarrow{AB}$ and $C \notin \overleftrightarrow{AB}$ and side S (contains C) and side T (doesn't contain C)

By Ax 3 there exist an isometry, f st

$f(A) = A$

$f(B) \in \overrightarrow{AB}$

$f(C) \in T$

By Thm 22.5 $f(S) = T$

Thm 22.5 DON'T NEED TO PROVE

Given lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ and side S of $\overleftrightarrow{AB}$ and isometry f Such that $f(\overleftrightarrow{AB}) = \overleftrightarrow{CD}$ then $f(S)$ is a side of $\overleftrightarrow{CD}$