

Theorem 29 (SAS): If a triangle has two sides and the included angle congruent to two sides and the included angle of another triangle, then the triangles are congruent

Given triangles $\triangle ABC$ and $\triangle DEF$ such that

- $\overline{AB} \cong \overline{DE}$, (1)

- $\overline{BC} \cong \overline{EF}$ and (2)

- $\angle ABC \cong \angle DEF$ (3)

Then

- $\underline{AB} = \underline{DE}$ because (1) (4)

- $\underline{BC} = \underline{EF}$ because (2) (5)

- $m(\angle ABC) = m(\angle DEF)$ because (3) (6)

By Axiom 3, there is an isometry f (7)

such that

- $f(B) = E$ (8)

- $f(A) \in \overline{ED}$ (9)

- $f(C)$ is on the same side of \overline{ED} as point F (10)

By Theorem 26 and lines (7, 4, 8, 9), $f(A) = D$ (11)

By Theorem 27 and lines (7, 6, 8, 9, 10), $f(C) \in \overline{EF}$ (12)

By Theorem 26 and lines (7, 2, 8, 12), $f(C) = F$ (13)

So $f(A) = D$, $f(B) = E$, $f(C) = F$ (by lines 11, 8, 13), and by theorem 18, $f(\triangle ABC) = \triangle DEF$

Therefore $\triangle ABC \cong \triangle DEF$