Theorem 18: For an isometry $f: E^2 \to E^2$ and distinct points A, B and $f(A) = A'$ and $f(B) = B'$

a) The isometric image of a line segment is a line segment: $f(\overline{AB}) = \overline{A'B'}$

Proof:

Let $f: E^2 \to E^2$ be ________________________, and let A, B be ________________________

Now, $f(\overline{AB}) = \{f(X) \mid X \in$ _______$\}$

And $\overline{A'B'} = \{X \mid$ _____ + ______ = ______$\}$

Let $Y \in f(\overline{AB})$

Then $Y = f(X)$ such that _________

So, $\underline{AX} + \underline{XB} = \underline{AB}$

Because f is an isometry, we know that

$\underline{AX} = \underline{f(A)f(X)} = \underline{A'Y}$, and $\underline{XB} =$ ____________ and $\underline{AB} =$ _____________________

Substituting in, we get that

______ + _______ = _________

And therefore ______ $\in \overline{A'B'}$

Thus $f(\overline{AB}) \subseteq \overline{A'B'}$

Let $Y \in \overline{A'B'}$

Then, by definition: ____ + ____ $= \underline{A'B'}$

Because f is an isometry, it is an onto function, and hence $Y =$ ________ for some point X

Because f is an isometry, we know

$\underline{A'Y} =$ _____ and $\underline{YB'} =$ _______ and $\underline{A'B'} =$ ________

Substituting in, we get that

______ + _______ = _________

And so _____ $\in \overline{AB}$, and hence $Y \in f(\overline{AB})$.

Hence, $\overline{A'B'} \subseteq f(\overline{AB})$

Therefore $\overline{A'B'} = f(\overline{AB})$

Theorem 19: The isometric image of a circle is a circle, specifically, for an isometry $f : E^2 \to E^2$ and a circle $C = \odot(A, r)$ with center A and radius r (where A is a point and r is a positive real number), then $f(C) = \odot(f(A), r) = \odot(A', r)$ where $f(A) = A'$.

Proof:

Let ____,

and let ______________________________________

Now, $f(C) =$ ________________________________

And $\odot(A', r) =$ _______________________________

Let $Y \in f(C)$

Then $Y =$ ____

____

Because *f* is an isometry, we know that

____

Substituting in, we get that

____

____

Thus $f(C) \subseteq \odot(A', r)$

Let $Y \in \odot(A', r)$

Then, by definition: ____________________________

Because *f* is an isometry, it is an onto function, and hence ______________________________

Because *f* is an isometry, we know

Substituting in, we get that

____

____

Hence, $\odot(A', r) \subseteq f(C)$

Therefore $f(C) = \odot(A', r)$