Geometry homework

Fill in the blanks to prove

**Theorem 17:** The composition of two isometries is an isometry, and hence congruence is transitive: if $S \cong T$ and $T \cong V$ then $S \cong V$

Proof:

Let $f: E^2 \to E^2$ and $g: E^2 \to E^2$ be isometries.

Then *f* and *g* are both ________________________, so by theorem 11, $g \circ f$ is _______________

Also, because they are isometries, Then *f* and *g* are both ________________________, so by theorem 12, $g \circ f$ is _______________

And, because they are isometries, Then *f* and *g* are both ________________________, so by theorem 13, $g \circ f$ is _______________

Let $A, B \in E^2$

Then $d(f(A), f(B)) =$ _________________ because ______ is an isometry. (1)

And $d(g(f(A)), g(f(B))) = d(\quad\quad , \quad\quad)$ because $g$ is an isometry. (2)

So, $d(g(f(A)), g(f(B))) = d(\quad\quad , \quad\quad) = d(\quad\quad , \quad\quad)$ by lines (2) and (1)

Therefore $g \circ f$ is an isometry, and the composition of any two isometries is an isometry.

Let $S \cong T$ and $T \cong V$,

Then there exist isometries $f: E^2 \to E^2$ and $g: E^2 \to E^2$ such that $f(S) = T$ and $g(T) = V$

Because *f* and *g* are isometries, we know ____________ is an isometry, and

$g(f(S)) = g($_____$) =$ ____

Thus, $S \cong V$