Geometry Axioms and Theorems

Definition: The **plane** is a set of points that satisfy the axioms below. We will sometimes write E^2 to denote the plane.

Axiom 1: There is a metric on the points of the plane that is a distance function, which we will denote $d : E^2 \times E^2 \to [0,\infty)$. Given points $A, B \in E^2$, then $d(A,B)$ is called the **distance** between the points *A* and *B*, and we also use the notation: $d(A,B) = \underline{AB}$. The distance function satisfies the conditions

a)	$d(A,B) = 0$ if and only if $A = B$	$\underline{AB} = 0$ if and only if $A = B$
b)	$d(A,B) = d(B,A)$	$\underline{AB} = \underline{BA}$
c)	If $A, B, C \in E^2$ then $d(A,B) + d(B,C) \geq d(A,C)$	$\underline{AC} \leq \underline{AB} + \underline{BC}$

Definition: Given two points $A, B \in E^2$, the line segment between them is defined to be the set: $\overline{AB} = \{X \in E^2 \mid \underline{AX} + \underline{XB} = \underline{AB}\}$

Q1: How can we use the definition of a segment to define a ray?

Q2: How can we use the definition of a segment and/or ray to define an infinite line?

Q3: If there are 4 points on a line, and in order they are *A*-*B*-*C*-*D*, what equation or equations would be true about the distances between those points?

Q4: If you were going to define a circle using the distance function, what would the definition be?