Geometry Axioms and Theorems

Definition: The **plane** is a set of points that satisfy the axioms below. We will sometimes write E^2 to denote the plane.

Axiom 1: There is a metric on the points of the plane that is a distance function, which we will denote $d : E^2 \times E^2 \to [0,\infty)$. Given points $A, B \in E^2$, then $d(A,B)$ is called the **distance** between the points *A* and *B*, and we also use the notation: $d(A,B) = \underline{AB}$. The distance function satisfies the conditions

a) $d(A,B) = 0$ if and only if $A = B$ — $\underline{AB} = 0$ if and only if $A = B$

b) $d(A,B) = d(B,A)$ — $\underline{AB} = \underline{BA}$

c) If $A,B,C \in E^2$ then $d(A,B) + d(B,C) \geq d(A,C)$ — $\underline{AC} \leq \underline{AB} + \underline{BC}$

Definition: Given two points $A, B \in E^2$, the line segment between them is defined to be the set: $\overline{AB} = \{X \in E^2 \mid \underline{AX} + \underline{XB} = \underline{AB}\}$

Q1: How can we use the definition of a segment to define a ray?

$\overrightarrow{AB} = \{X \in E^2 \mid \underline{AX} + \underline{XB} = \underline{AB} \quad \text{or} \quad \underline{AB} + \underline{BX} = \underline{AX}\}$

Q2: How can we use the definition of a segment and/or ray to define an infinite line?

*Write a definition—due Weds 9/11

Q3: If there are 4 points on a line, and in order they are *A-B-C-D*, what equation or equations would be true about the distances between those points?

$\underline{AB} + \underline{BC} + \underline{CD} = \underline{AD}$

Q4: If you were going to define a circle using the distance function, what would the definition be?

Given a (center) point $A \in E^2$ and a (radius) distance $r \in [0,\infty)$, the circle with center *A* and radius *r* is defined to be $\{X \in E^2 \mid \underline{AX} = r\}$ and we will sometimes denote it $\odot(A,r)$

Theorem (*try to prove for Wednesday*—may be really easy, may be impossible: this is a test to see if our definition of "order" is good enough): Given $A - B - C - D$ (which means, by definition, $\underline{AB} + \underline{BC} + \underline{CD} = \underline{AD}$), prove that $\underline{AB} + \underline{BC} = \underline{AC}$