Thm 1 Proof. Let $A, B \in E^2$.

Let $X \in \overline{AB}$

Then, by defn. of $\overline{AB}$, $\underline{AX} + \underline{XB} = \underline{AB}$.

Now, by Axiom 1 (part b)

$$\underline{AX} = \underline{XA}$$
$$\underline{XB} = \underline{BX}$$
$$\underline{AB} = \underline{BA}.$$

So $\underline{XA} + \underline{BX} = \underline{BA}$

and $\underline{BX} + \underline{XA} = \underline{BA}$

so, by definition of $\overline{BA}$, $X \in \overline{BA}$

→ Thus $\overline{AB} \subseteq \overline{BA}$

Similarly, we can show $\overline{BA} \subseteq \overline{AB}$

And therefore $\overline{AB} = \overline{BA}$

☆ To show subsets: $S \subseteq T$

You start with: Let $X \in S$

then do stuff to prove $X \in T$

and you conclude $S \subseteq T$

☆ If you can do everything the same, but change the names of the points/variables and get a new result, you are allowed to use the word "similarly" instead of rewriting it all.

☆ To prove two sets are equal $S = T$

You have to prove both $S \subseteq T$ and $T \subseteq S$

Thm 3a proof

Given points with order A-B-C-D

then by definition

$$AB + BC + CD = AD \qquad (1)$$

By Ax 1 (c) we know

$$AB + BC \geq AD \qquad (2)$$

could be > or =

this wording usually means we are trying to get a contradiction

Suppose $AB + BC > AC$

then $AB + BC + CD > AC + CD$

add same to both sides

so $AD > AC + CD$

substitute line (1)

But by Ax 1 (c) $AD \leq AC + CD$
which is a contradiction.

Hence $AB + BC > AC$ is false

so $AB + BC = AC$ because of (2)

By definition of $\overline{AC}$, $B \in \overline{AC}$ ∎

Other hints/notes:

Theorem 2's proof is very similar to theorem 1's proof

Theorem 3b's proof is very similar to 3a's proof.

About the corollary to Thm 3:

Given $A-B-C-D$

then $D-C-B-A$ (by thm 2)

so $C \in \overline{DB}$ and $C \in \overline{DA}$ (thm 3)

and $C \in \overline{BD}$ and $C \in \overline{AD}$ (thm 1)

Theorem 4 can be proved using Thms 1–3.

You don't need any new algebra.

Theorem 5 proof:

$$\overleftrightarrow{AC} = \{X \in E^2 \mid {}^{1}\ \overline{AX} + \overline{XC} = \overline{AC} \text{ OR } {}^{2}\ \overline{AC} + \overline{CX} = \overline{AX} \text{ OR } {}^{3}\ \overline{XA} + \overline{AC} = \overline{XC}\}$$

$$\overleftrightarrow{AB} = \{X \in E^2 \mid {}^{a}\ \overline{AX} + \overline{XB} = \overline{AB} \text{ OR } {}^{b}\ \overline{AB} + \overline{BX} = \overline{AX} \text{ OR } {}^{c}\ \overline{XA} + \overline{AB} = \overline{XB}\}$$

If $B \in \overleftrightarrow{AC}$, then

case 1	case 2	case 3
1 $\overline{AB} + \overline{BC} = \overline{AC}$	2 $\overline{AC} + \overline{CB} = \overline{AB}$	3 $\overline{BA} + \overline{AC} = \overline{BC}$
and	and	so
b $\overline{AB} + \overline{BC} = \overline{AC}$	a $\overline{AC} + \overline{CB} = \overline{AB}$	c $\overline{CA} + \overline{AB} = \overline{CB}$
means	means	(by ax 1)
$C \in \overleftrightarrow{AB}$	$C \in \overleftrightarrow{AB}$	so $C \in \overleftrightarrow{AB}$

therefore $C \in \overleftrightarrow{AB}$

note:

to prove by cases,
each case has to lead to
either the conclusion or a contradiction

HW: Write proof of 3b to turn in
and
Prove theorem 4. (discussion)