Theorem 32

a. Given $\triangle ABC$ such that $\overline{BA} \cong \overline{BC}$

angle

By Thm. 28 $\angle ABC \cong \angle CBA$

$\triangle ABC$ and $\triangle CBA$ satisfy SAS:

$\angle ABC \cong \angle CBA$

first side of angle $\overline{AB} \cong \overline{CB}$

second side of angle $\overline{BC} \cong \overline{BA}$

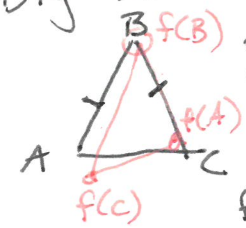
so

$\triangle ABC \cong \triangle CAB$

by CPCTC (Corresponding parts of congruent triangles are congruent)

$\angle BAC \cong \angle ACB$ $\square$

Big idea:



you could make an isometry

$f(B) = B$

$f(A) \in \overrightarrow{BC}$

$f(C)$ on side of $\overleftrightarrow{BC}$ including A,

We could use Thms 26 & 27 to get $f(A) = C$

$f(C) = A$.

then $f(\triangle ABC) = \triangle CBA$.

Wait! That was the proof of SAS - Let's just use SAS!

b. Given $\triangle ABC$ such that

$\angle BAC \cong \angle BCA$

By thm 1 $\overline{AC} \cong \overline{CA}$

Side

$\triangle ABC$ and $\triangle CBA$ satisfy ASA

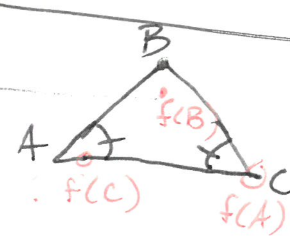
$\overline{AC} \cong \overline{CA}$

$\angle BAC \cong \angle BCA$ angle at first endpoint

$\angle BCA \cong \angle BAC$ angle at second endpoint

so, $\triangle ABC \cong \triangle CBA$

Because CPCTC, $\overline{AB} \cong \overline{CB}$ $\square$



Make the isometry like this

$f(A) = C$

$f(C) \in \overrightarrow{CA}$

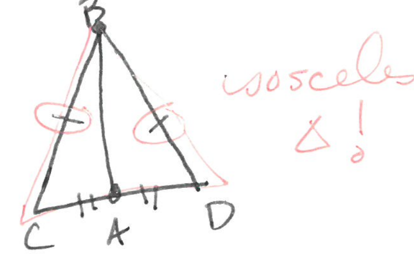
$f(B) \in$ side of $\overleftrightarrow{AC}$ with B

This looks like the Proof of ASA!

Thm 33.

Given $\triangle ABC$, $\triangle ABD$ such that $A \in \overline{CD}$ and
$\overline{AC} \cong \overline{AD}$, $\overline{BC} \cong \overline{BD}$

Pictures help!



$\triangle BCD$ is a triangle, and by thm 32a, because $\overline{BC} \cong \overline{BD}$,

we get $\angle BCD \cong \angle BDC$.

Now $\angle BCD = \angle BCA$

and $\angle BDC = \angle BDA$
($A \in \overline{CD}$: two names for the same angle)

So $\triangle ABC$ and $\triangle ABD$ satisfy SAS:

$\overline{BC} \cong \overline{BD}$

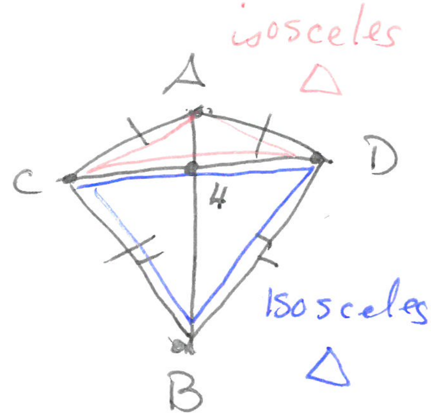
$\overline{CA} \cong \overline{DA}$

$\angle BCA \cong \angle BDA$. $\square$

Thm 34

Given $\triangle ABC$, $\triangle ABD$ such that
$\overline{AB} \cap \overline{CD} = H$ and $H \neq A$, $H \neq B$.
and $\overline{AC} \cong \overline{AD}$
$\overline{BC} \cong \overline{CD}$
$\overline{AB} \cong \overline{AB}$

Guiding Picture



By thm 32a, because
$\overline{AC} \cong \overline{AD}$,
we get $\angle ACD \cong \angle ADC$
By thm 32a, because $\overline{BC} \cong \overline{BD}$
we get $\angle BCD \cong \angle BDC$

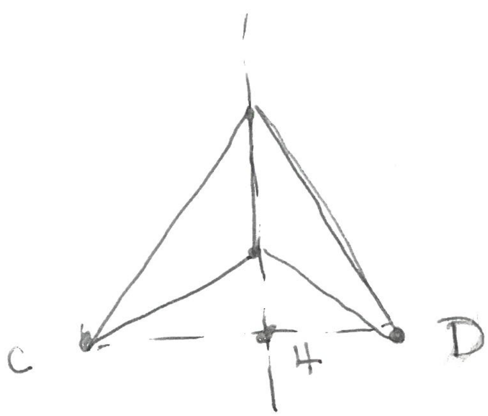
By Ax 4, we know
$m(\angle ACB) = m(\angle ACD) + m(\angle DCB)$
$m(\angle ADB) = m(\angle ADC) + m(\angle CDB)$

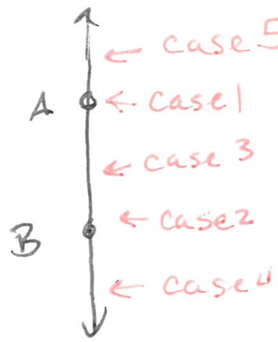
By substitution, we get
$$m(\angle ACB) = m(\angle ACD) + m(\angle DCB) = m(\angle ADC) + m(\angle CDB)$$
$$= m(\angle ADB)$$

So $\triangle ABC$ and $\triangle DEF$ satisfy SAS:
$\angle ACB \cong \angle ADB$
$\overline{AC} \cong \overline{AD}$
$\overline{CB} \cong \overline{DB}$

Thm 36 proof

Given $\triangle ABC$ & $\triangle ABD$, where C & D are on opp. sides of $\overleftrightarrow{AB}$ s.t. $\overline{AC} \cong \overline{AD}$ and $\overline{BC} \cong \overline{BD}$





Let H be the intersection of $\overleftrightarrow{AB}$ and $\overline{CD}$

Case 1: $H = A$ by thm 33 and SAS, $\triangle ABC \cong \triangle ABD$

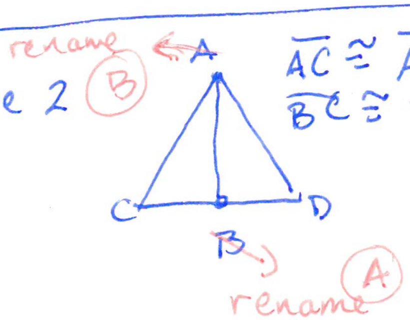
Case 2: $H = B$ Rename $A \to B$ and $B \to A$, Because A & B have the same given properties *
by thm 33, and SAS, $\triangle ABC \cong \triangle ABD$

Case 3: $H \in \overline{AB}$, but $H \neq A, B$ by thm 34 and SAS, $\triangle ABC \cong \triangle ABD$

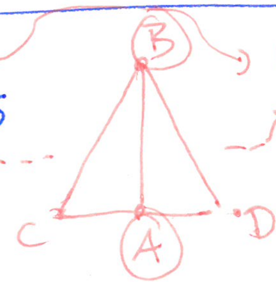
Case 4: $H \in \overrightarrow{AB}$, but $H \notin \overline{AB}$ by thm 35 and SAS $\triangle ABC \cong \triangle ABD$

Case 5: $H \in \overrightarrow{BA}$, but $H \notin \overline{AB}$ Rename $A \to B$, $B \to A$
now the properties fit Thm 35
so $\triangle ABC \cong \triangle ABD$

Case 2 (B)



$\overline{AC} \cong \overline{AD}$
$\overline{BC} \cong \overline{BD}$



$\overline{BC} \cong \overline{BD}$
$\overline{AC} \cong \overline{AD}$
Satisfies Thm 33
so $\triangle ABC$, $\triangle ABD$ share SAS
so $\triangle BAC$, $\triangle BAD$ share SAS ✓