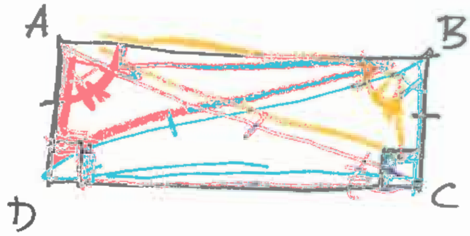
Given



quad ABCD such that $\overline{AD} \cong \overline{BC}$

and $m\angle ADC = m\angle BCD = 90^\circ$

By SAS, because $\overline{AD} \cong \overline{BC}$

$\overline{DC} \cong \overline{CD}$

$\angle ADC \cong \angle BCD$

$\triangle ADC \cong \triangle BCD$

so: $\angle CAD \cong \angle DBC$

$\angle ACD \cong \angle BDC$

$\overline{AC} \cong \overline{BD} \rightarrow$ diagonals are congruent!

By SSS, because: $\overline{BD} \cong \overline{AC}$

$\overline{AD} \cong \overline{BC}$

$\overline{AB} \cong \overline{AB}$

So $\triangle ABD \cong \triangle BAC$

Starting with the givens, we showed that the two triangles in purple and blue are congruent. We can use these triangles to prove the diagonals are congruent

$\triangle ABD \cong \triangle BAC$

so $\angle ADB \cong \angle BCA$

and $\angle ABD \cong \angle BAC$

and $\angle BAD \cong \angle ABC$

↳ so $m\angle BAD = m\angle ABC$

By Q1

$m\angle ADC + m\angle BCD + m\angle BAD + m\angle ABC = 360$

$90 + 90 + 2m\angle BAD = 360^\circ$

↓ algebra (easy)

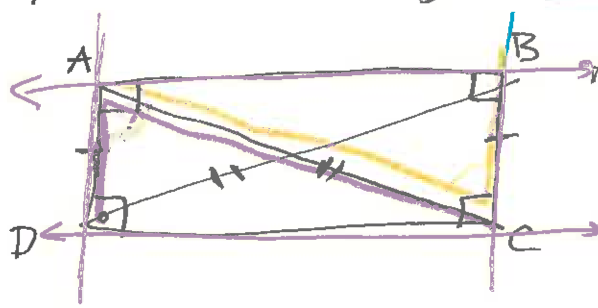
$m\angle BAD = 90^\circ$

so $m\angle ABC = 90^\circ$

it's a rectangle!

Next we got two more congruent triangles and used them along with the sum of angles in a quadrilateral to prove that the it is a rectangle

We know so far



$m\angle DAB + m\angle ABC = 90 + 90 = 180^\circ$

(think $\overleftrightarrow{AB}$ transversal)

so $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$

$m\angle ADC + m\angle DAB = 90+90 = 180$

(think $\overleftrightarrow{AD}$ transversal)

so $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

$\overline{AD} \parallel \overline{BC}$

$\overline{AC}$ is a transversal

so by Ax 7 & 54

$\angle DAC \cong \angle BCA$

By SAS $\angle DAC \cong \angle BCA$

$\overline{AD} \cong \overline{BC}$, $\overline{AC} \cong \overline{CA}$

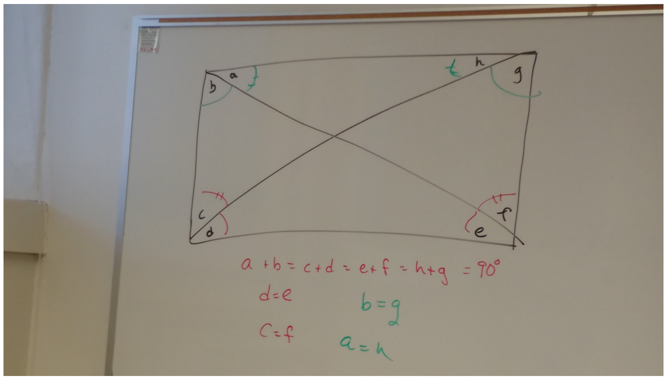
so $\triangle DAC \cong \triangle BCA$

so $\overline{AB} \cong \overline{CD}$

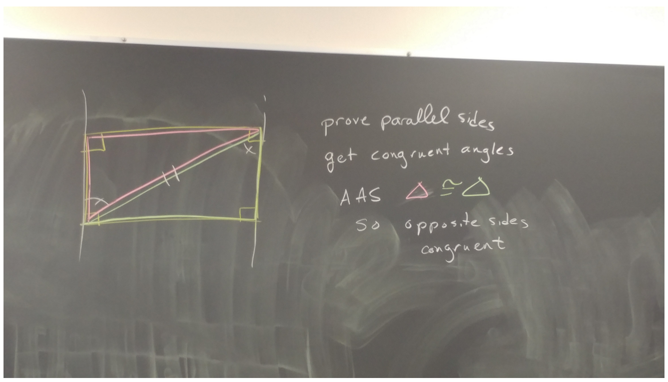
Then we used the right angles to prove it is a parallelogram.

Then we used the parallel sides to prove that some angles were equal (Axiom 7), and hence the purple and orange triangles here are congruent

Those triangles let us show the other pair of sides are congruent

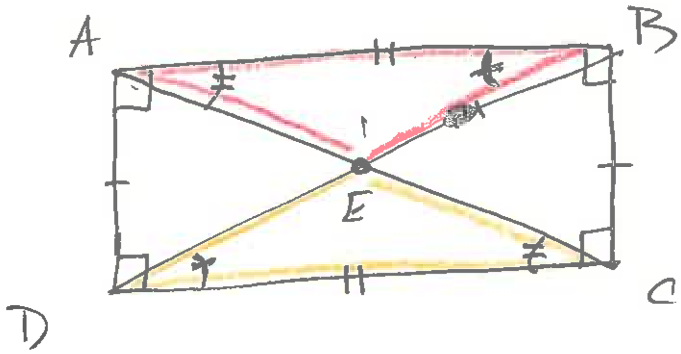


On the first page, we proved a lot of angles were equal, but not the ones we needed to know to prove that triangles ADC and CBA are congruent, so we needed to know and use parallel sides.



If you had another rectangle (but you didn't know that any sides were congruent), you could prove the sides were parallel, and then get congruent angles from Axiom 7, and use that to prove that the opposite sides were congruent.

Given:



$\overline{AC} \cong \overline{BD}$

Let $\overline{AC} \cap \overline{BD} = E$

and $\overline{AB} \parallel \overline{DC}$

so by Ax 7 (Thm 54)

$\angle ABE \cong \angle CDE$

and $\angle BAE \cong \angle DCE$

and $\overline{AB} \cong \overline{CD}$

By: ASA

$\triangle DCE \cong \triangle BAE$

so $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$

E is the midpoint of both diagonals

The last thing we did in class, was to take that same quadrilateral we have been working on since the beginning and prove that the red and yellow triangles here are congruent, and so the intersection of the two diagonals is the midpoint of both diagonals.

If you understand all of these steps, you should be able to use them to prove a lot more of the quadrilateral conjectures.