Thm 17a. If $d(A,B) = d(C,D)$ and there is an isometry, $f$, such that $f(A) = C$ an $f(B) \in \overrightarrow{CD}$ then $f(B) = D$.

Thm 18a. If $m(\measuredangle ABC) = m(\measuredangle DEF)$ and there is an isometry, $f$, such that $f(B) = E$, $f(A) \in \overrightarrow{ED}$ and $f(C)$ is on the same side of $\overleftrightarrow{ED}$ as $F$ then $f(C) \in \overrightarrow{EF}$

Nic Persoon

Theorem 17:

If two segments have the same length then they are congruent

Proof:

- Let there be two segments $\overline{AB}$ and $\overline{CD}$ where $d(A,B) = d(C,D)$
- By axiom 3, there is an isometry $f$ such that $f(A) = C$ and $f(B) \in \overrightarrow{CD}$
- Let $f(A) = A' = C$ and $f(B) = B' \in \overrightarrow{CD}$
- Since $B' \in \overrightarrow{CD}$, we know that:
  - $B' = D$, $B' \in \overline{CD}$ or $D \in \overline{CB'}$
- Case 1: $B' \in \overline{CD}$
  - By definition of a line segment, $d(C,B') + d(B',D) = d(C,D)$
  - Since $A' = C$, we use substitution:
  - $d(A',B') + d(B',D) = d(C,D)$
  - Since $d(A,B) = d(C,D)$, then $d(A',B') + d(B',D) = d(A,B)$
  - By definition of an isometry, we know $d(A,B) = d(A',B')$
  - So $d(A,B) + d(B',D) = d(A,B)$
  - Then $d(B',D) = 0$
  - Thus $B' = D$
- Case 2: $D \in \overline{CB'}$
  - By definition of a line segment, $d(C,D) + d(D,B') = d(C,B')$
  - Since $A' = C$, we use substitution:
  - $d(C,D) + d(D,B') = d(A',B')$
  - Since $d(A,B) = d(C,D)$, then $d(A,B) + d(D,B') = d(A',B')$
  - By definition of isometry, we know $d(A,B) = d(A',B')$
  - So $d(A,B) + d(D,B') = d(A,B)$
  - Then $d(D,B') = 0$
  - Thus $B' = D$

Want to prove

$d(f(B), A) = 0$ if $\Downarrow$ Ax 1 then $f(B) = A$

Ax 1 $\to$ $d(A,B) = d(B,A)$

Ax 1 $\to$ $d(A',B') = d(B',A')$

We know $A' = B$

$B' \in \overrightarrow{BA}$

$d(A,B) = d(A',B')$ (1) because $f$ preserves distances

$d(A,B) = d(A,A')$

$\boxed{A' = B}$

$B' \in \overrightarrow{BA}$ means

$d(B,B') + d(B',A) = d(B,A)$ $\longrightarrow$ case 2 ...

OR

Case 1

$d(B,A) + d(A,B') = d(B,B')$

$B = A'$

$d(A,B) + d(A,B') = d(A',B')$ (1)

$d(A,B) + d(A',B') = d(A',B)$

$-d(A,B)$ $\quad$ $-d(A,B)$

So $d(A,B') = 0$

So $A = B'$

Case 3

$C'$ is outside $\angle DEF$

$C' \notin \measuredangle DEF$

$$\measuredangle C'ED + \measuredangle DEF = \measuredangle C'EF$$

$$\Downarrow$$

$$m \measuredangle C'ED = 0 \quad \times$$

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Thus $\measuredangle ABC \cong \measuredangle FED \cong \measuredangle DEF$

so $\measuredangle ABC \cong \measuredangle DEF$ by thm15