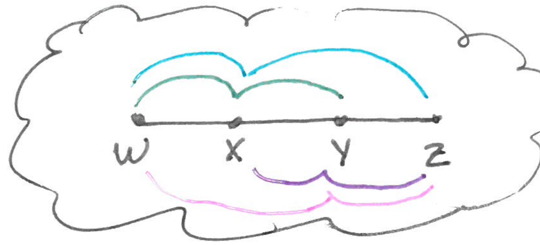

Thinking

Theorem 5

Equation Number

Given: $w - x - y - z$

by defn.: $d(w,x) + d(x,y) + d(y,z) = d(w,z)$ (1)

by Ax 1:

$d(w,y) \le d(w,x) + d(x,y)$ (2)

$d(w,z) \le d(w,x) + d(x,z)$ (3)

$d(x,z) \le d(x,y) + d(y,z)$ (4)

$d(w,z) \le d(w,y) + d(y,z)$ (5)

$$d(w,z) \underset{(3)}{\le} d(w,x) + \underline{d(x,z)} \underset{(4)}{\le} d(w,x) + d(x,y) + d(y,z) \underset{(1)}{=} d(w,z)$$

so

$$d(w,z) = \underbrace{d(w,x) + d(x,z) = d(w,x) + d(x,y) + d(y,z)} = d(w,z)$$

so $d(w,z) = d(w,x) + d(x,z)$ (6)

and $d(w,x) + d(x,z) = d(w,x) + d(x,y) + d(y,z)$

$-d(w,x) \qquad -d(w,x)$

$d(x,z) = d(x,y) + d(y,z)$ (7)

$$d(w,z) \underset{(5)}{\le} d(w,y)+d(y,z) \underset{(2)}{\le} d(w,x)+d(x,y)+d(y,z) \underset{(1)}{=} d(w,z)$$

so

$$d(w,z) = d(w,y)+d(y,z) = d(w,x)+d(x,y)+d(y,z) = d(w,z)$$

so $\quad d(w,z) = d(w,y)+d(y,z) \qquad (8)$

and $\quad d(w,y)+d(y,z) = d(w,x)+d(x,y)+d(y,z)$

$\quad -d(y,z) \qquad\qquad -d(y,z)$

so $\quad d(w,y) = d(w,x)+d(x,y) \qquad (9)$

$d(w,z) = d(w,x)+d(x,z) \qquad (6)$

so by thm 4 $\quad x \in \overleftrightarrow{wz}$, $w \in \overleftrightarrow{xz}$, $z \in \overleftrightarrow{wx}$

$d(x,z) = d(x,y)+d(y,z) \qquad (7)$

so by thm 4 $\quad x \in \overleftrightarrow{yz}$, $y \in \overleftrightarrow{xz}$, $z \in \overleftrightarrow{xy}$

$d(w,z) = d(w,y)+d(y,z) \qquad (8)$

so by thm 4 $\quad w \in \overleftrightarrow{yz}$, $y \in \overleftrightarrow{wz}$, $z \in \overleftrightarrow{yw}$

$d(w,y) = d(w,x)+d(x,y) \qquad (9)$

so by thm 4 $\quad w \in \overleftrightarrow{xy}$, $x \in \overleftrightarrow{wy}$, $y \in \overleftrightarrow{wx}$

so each point lies on the line defined by any two of the other three points