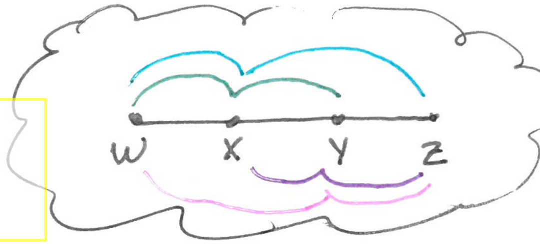
For Thm 3

Proving thm 3 is like thm 5 but you only do part of the thm 5 work

~~Theorem 5~~



Thinking

Equation Number

Given: $w - x - y - z$

by defn.: $d(w,x) + d(x,y) + d(y,z) = d(w,z)$ (1)

by Ax 1:

$d(w,y) \le d(w,x) + d(x,y)$ (2)

$d(w,z) \le d(w,x) + d(x,z)$ (3)

$d(x,z) \le d(x,y) + d(y,z)$ (4)

$d(w,z) \le d(w,y) + d(y,z)$ (5)

$d(w,z) \underset{(3)}{\le} d(w,x) + \underline{d(x,z)} \underset{(4)}{\le} d(w,x) + d(x,y) + d(y,z) \underset{(1)}{=} d(w,z)$

so

$d(w,z) = d(w,x) + d(x,z) = d(w,x) + d(x,y) + d(y,z) = d(w,z)$

so $d(w,z) = d(w,x) + d(x,z)$ (6)

and $d(w,x) + d(x,z) = d(w,x) + d(x,y) + d(y,z)$

$-d(w,x) \qquad -d(w,x)$

You don't need line 7 or line 9, and when you finish line 8, then you go straight to the conclusion, you don't use theorem 4

$d(x,z) = d(x,y) + d(y,z)$ (7)

3

Thm 3:

So (by 6), $x \in \overline{wz} \subseteq \overleftrightarrow{wz}$

So (by 8), $y \in \overline{wz} \subseteq \overleftrightarrow{wz}$, so $w, x, y, z \in \overleftrightarrow{wz}$

so they are collinear

$d(w,z) \le d(w,y) + d(y,z) \le d(w,x) + d(x,y) + d(y,z) = d(w,z)$

(5) (2) (1)

so

$d(w,z) = d(w,y) + d(y,z) = d(w,x) + d(x,y) + d(y,z) = d(w,z)$

so $d(w,z) = d(w,y) + d(y,z)$ (8)

and $d(w,y) + d(y,z) = d(w,x) + d(x,y) + d(y,z)$

After line 8, you jump straight to showing that points are on lines (see bottom of page 1). The rest of this page isn't used in theorem 3.

so $d(w,y) = d(w,x) + d(x,y)$ (9)

$d(w,z) = d(w,x) + d(x,z)$ (6)

so by thm 4 $x \in \overleftrightarrow{wz}$, $w \in \overleftrightarrow{xz}$, $z \in \overleftrightarrow{wx}$

$d(x,z) = d(x,y) + d(y,z)$ (7)

so by thm 4 $x \in \overleftrightarrow{yz}$, $y \in \overleftrightarrow{xz}$, $z \in \overleftrightarrow{xy}$

$d(w,z) = d(w,y) + d(y,z)$ (8)

so by thm 4 $w \in \overleftrightarrow{yz}$, $y \in \overleftrightarrow{wz}$, $z \in \overleftrightarrow{yw}$

$d(w,y) = d(w,x) + d(x,y)$ (9)

so by thm 4 $w \in \overleftrightarrow{xy}$, $x \in \overleftrightarrow{wy}$, $y \in \overleftrightarrow{wx}$

so each point lies on the line defined by any two of the other three points