Theorem 4: If 3 points X, Y, Z satisfy a segment equation $d(X,Y) + d(Y,Z) = d(X,Z)$, then each point lies on the line defined by the other two. IE $X \in \overleftrightarrow{YZ}$, $Y \in \overleftrightarrow{XZ}$ & $Z \in \overleftrightarrow{XY}$

Part 1: $\overleftrightarrow{YZ} = \{W \mid d(Z,Y) + d(Y,W) = d(Z,W)$ or $d(Y,Z) + d(Z,W) = d(Y,$
or $d(Y,W) + d(W,Z) = d(Y,Z)\}$

we know $d(X,Y) + d(Y,Z) = d(X,Z)$

$d(Z,Y) + d(Y,W) = d(Z,W)$

$d(Z,Y) + d(Y,X) = d(Z,X)$

so $X \in \overleftrightarrow{YZ}$

Part 2: $\overleftrightarrow{XZ} = \{W \mid d(Z,X) + d(X,W) = d(Z,W)$ or $d(X,Z) + d(Z,W) = d($
or $d(Y,W) + d(W,Z) = d(X,Z)\}$

we know $d(X,Y) + d(Y,Z) = d(X,Z)$

$d(X,W) + d(W,Z) = d(X,Z)$

so $Y \in \overleftrightarrow{XZ}$

PART 3: $\overleftrightarrow{XY} = \{W \mid d(Y,X) + d(X,W) = d(Y,W)$ or $d(X,Y) + d(Y,W) = d(X,$
or $d(X,W) + d(W,Y) = d(X,Y)\}$

we know $d(X,Y) + d(Y,Z) = d(X,Z)$

$d(X,Y) + d(Y,W) = d(X,W)$

so $Z \in \overleftrightarrow{XY}$