A *function* $f : A \to B$ is a (often infinite) set of ordered pairs $\{(x, f(x)) \mid x \in A, f(x) \in B\}$, such that for any element $a \in A$, there is a unique ordered pair $(a, f(a))$, where *a* is the first coordinate of the ordered pair. Informally, we say that *f* is a rule that assigns to each element of *A* a unique element of *B*. The uniqueness property can be proved by proving that, for any pair of ordered pairs $(a, f(a)), (a', (f(a'))$, if $a = a'$ then $f(a) = f(a')$.

A function $f : A \to B$ is *onto* if for every $b \in B$ there is an element $a \in A$ such that *f(a)=b*.

A function $f : A \to B$ is *1-to-1* if for any $a, a' \in A$, if $f(a) = f(a')$ then $a = a'$. The informal notion of this is the contrapositive of this definition: if two elements of A are different, then their images are different.

There is a *1-1 correspondence* between sets A and B if there is a 1-1, onto function $f : A \to B$.

FT1: Let $f(x) = x - 3$ for all $x \in \mathbb{R}$. Prove (using the above definitions) that *f* is a 1-1, onto function $f : \mathbb{R} \to \mathbb{R}$.

FT 2: Let $f(x) = -x$ for all $x \in \mathbb{R}$. Prove (using the above definitions) that *f* is a 1-1, onto function $f : \mathbb{R} \to \mathbb{R}$.

FT3: Let $f : A \to B$ be a 1-1 onto function, then there exists a function $f^{-1} : B \to A$ such that $f^{-1}(f(a)) = a$ and $f(f^{-1}(b)) = b$ for all $a \in A$ and $b \in B$.

FT4: Let $f : A \to B$ and $g : B \to C$ be 1-1 functions, then $g(f) : A \to C$ is a 1-1 function.

FT5: Let $f : A \to B$ and $g : B \to C$ be onto functions, then $g(f) : A \to C$ is an onto function.

A function $f : A \to B$, where distance and angle measurement is defined in both sets *A* and *B*, is called an isometry if $d(a, a') = d(f(a), f(a'))$ and if $m(\angle a' a a'') = m(f(a') f(a) f(a''))$ for all $a, a', a'' \in A$.

In the real numbers, distance is defined to be the absolute value of the difference between the numbers.

FT 6: Let $f : A \to B$ and $g : B \to C$ be isometries, then $g(f) : A \to C$ is an isometry

FT 7: Let $f : A \to B$ and $f^{-1} : B \to A$ be inverse functions, then $f^{-1} : B \to A$ is 1-1 and onto
.

FT 8: Let $f : A \to B$ be an isometry, and let $f^{-1} : B \to A$ be its inverse function, then f^{-1} is an isometry.