25. Prove that the composition of two 1-1 functions is 1-1.

Proof: Given functions $f : X \to Y$ and $g : Y \to Z$ that are both 1-1 functions.

This means:

If $f(a) = f(b)$ for any $a, b \in X$ then $a = b$	If $g(u) = g(v)$ for any $u, v \in Y$ then $u = v$

There is a function $g \circ f : X \to Z$

Suppose $g \circ f(a) = g \circ f(b)$ for some $a, b \in X$

Then $g(f(a)) = g(f(b))$ and if I rename $u = f(a)$ and $v = f(b)$, then $g(u) = g(v)$

Since *g* is 1-1, we know that $u = v$

So $f(a) = f(b)$

Since *f* is 1-1, we know that $a = b$

This if $g \circ f(a) = g \circ f(b)$ for some $a, b \in X$, then $a = b$.

This proves that $g \circ f$ is 1-1.

26. Prove that the composition of two onto functions is onto.

Proof: Given functions $f : X \to Y$ and $g : Y \to Z$ that are both onto functions.

This means

If *u* is any element in *Y*, then somewhere in *X* there is an element *a* that maps to *u* (so $f(a) = u$)	If *t* is any element in *Z*, then somewhere in *Y* there is an element *v* that maps to *t* (so $g(v) = t$)

There is a function $g \circ f : A \to Z$	(because the codomain of *f* is the domain of *g*.)
Let $r \in Z$	(This means: pick any element in *Z* and name it *r*).
Because *g* is onto, there exists $w \in Y$ such that $g(w) = r$	This means: somewhere in *Y* there is an element that maps to *r*; let's name it *w*.
Now because $w \in Y$ and *f* is onto, there exists $b \in X$ such that $f(b) = w$	This means: somewhere in *X* there is an element that maps to *w*, let's name it *b*.

So, $g(f(b)) = g(w) = r$

We have shown that given any element *r*, there exists and element $b \in X$ such that $g \circ f(b) = r$, and hence $g \circ f$ is onto.