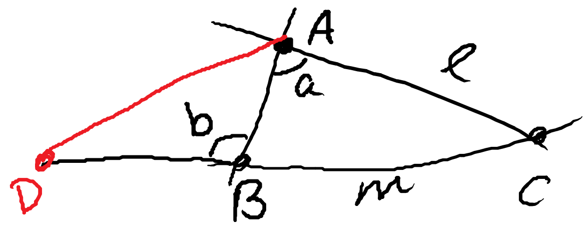
Theorem 32 (If AIA then parallel): Given two lines and a transversal line that intersects both lines, if the alternate interior angles are congruent, then the lines are parallel.

> Given: lines l and m that both intersect line t, with alternate interior angles $\angle a$ and $\angle b$ at intersection points $A = l \cap t$ and $B = m \cap t$ respectively, such that $\angle a \cong \angle b$
>
> To prove: l and m do not intersect.



Proof: Suppose l and m do intersect at point C

There are 2 cases:
Case 1: C is on the same side of t as $\angle a$
Case 2: C is on the same side of t as $\angle b$

By theorem ______, there exists a point D on line m such that D is on the opposite side of t from C, and $\overline{AC} \cong \overline{BD}$

Then ΔABD and ΔBAC are triangles, and

- $\overline{BA} \cong \overline{AB}$
- $\angle a \cong \angle b$
- $\overline{AC} \cong \overline{BD}$

So, by theorem _______ $\Delta ABD \cong \Delta BAC$
And hence, $\angle BAD \cong \angle ABC$

By axiom _____ $m\angle b + m\angle ABC = 180°$
By substitution $m\angle a + m\angle BAD = 180°$
Let E be a point on line l that is on the opposite side of t from C,
Then by axiom _______ $m\angle a + m\angle BAE = 180°$
By combining the equations $m\angle a + m\angle BAD = 180°$ and $m\angle a + m\angle BAE = 180°$,
we get that $\angle BAD \cong \angle BAE$
By theorem _____, since the angles share side $\overrightarrow{BA}$ and lie on the same side of t, $\overrightarrow{AD} = \overrightarrow{AE}$ and $D \in \overleftrightarrow{AE} = l$
Thus, $C, D \in l \cap m$ which contradicts theorem _____.

So we know that l and m cannot intersect in a point on the same side of t as $\angle a$

Case 2: *C* is on the same side of *t* as $\angle b$

By theorem ______, there exists a point *D* on line ____such that …

Then ΔABD and ΔBAC are triangles, and

So, by theorem _______ , we get_____________

And hence, _________

By axiom _____, ___________+________=_______

By substitution _______________

Let *E* be a point on line ______ that is _______________________

Then by axiom _______, we get _______________.

By combining the equations _______________________ and______________________,

we get that ___________________

By theorem _____, since …

Thus, _________________which contradicts theorem _____.

So we know that *l* and *m* cannot intersect at a point on the same side of *t* as $\angle b$, and since all points in the plane are on *t* or on one of the two sides of *t*, therefore *l* and *m* cannot intersect, and are parallel.