Definition: Two geometric objects are ***distinct*** if they are not equal as point sets (they do not have all the same points as each other). For the purposes of this definition we will treat a point as a set consisting of a single point. Two geometric objects ***intersect*** if they have at least one point in common.

Definition: Two **segments** are **congruent** if they have the same *length*. Two **angles** are **congruent** if they have the same *angle measure*. Two **triangles** are congruent if they have respectively the **same side lengths and angle measures**.

Theorem 1 (Sides of Congruent Angles): If two congruent angles share a side (ray) and the other sides (rays) lie on the same side of the shared side, then their other sides are also shared. **I.e.** If $\angle ABC \cong \angle ABD$ and C and D lie on the same side of $\overleftrightarrow{AB}$ then $\overrightarrow{AC} = \overrightarrow{AD}$.

Theorem 2 (Endpoints of Congruent Segments): On a ray, there is at most one point at a given distance from the endpoint of the ray. **I.e.** If $C \in \overrightarrow{AB}$ and $\overline{CA} \cong \overline{BA}$ then $C = B$.

Theorem 3 (Line intersection): Any two distinct lines intersect in at most one point.

Theorem 4: The isometric image of a line segment is a line segment.

Theorem 5: The isometric image of a line is a line.

Definition: A ***circle*** with a given center and radius length is the set of all points whose distance from the center is equal to the given length.

Theorem 6: The isometric image of a circle is a circle with the same radius.

Theorem 7: The composition of two 1-1 functions is 1-1.

Theorem 8: The composition of two onto functions is onto.

Theorem 9: The composition of two isometries is an isometry.

Theorem 10 (angles can be duplicated): Given a ray ($\overrightarrow{AB}$) and a side of the line containing a ray, and an angle, there exists another ray ($\overrightarrow{AC}$) with the same starting point as the given ray, and on the given side, such that the angle between the rays ($\angle BAC$) is congruent to the given angle.

Theorem 11: (segments can be duplicated): Given a ray starting at a given point (A), and any distance, there exists another point (B) that lies on the ray such that the segment between the points ($\overline{AB}$) is equal to that distance.

Theorem 12: If two sides of one triangle are congruent to two corresponding sides of another triangle, and the angles contained by those sides congruent, then there is an isometry that maps one triangle onto the other.

> Given $\angle ABC \cong \angle DEF$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$;
>
> Describe an isometry that maps B to E, then one that maps A' to a point on $\overrightarrow{ED}$, and finally one that maps C to the same side of $\overleftrightarrow{DE}$ as F.
> Use the isometries and earlier theorems to prove that the image of the first triangle is equal to the second

Theorem 13 (SAS): If two sides of one triangle are congruent to two corresponding sides of another triangle, and the angles contained by those sides are congruent, then the triangles are congruent.