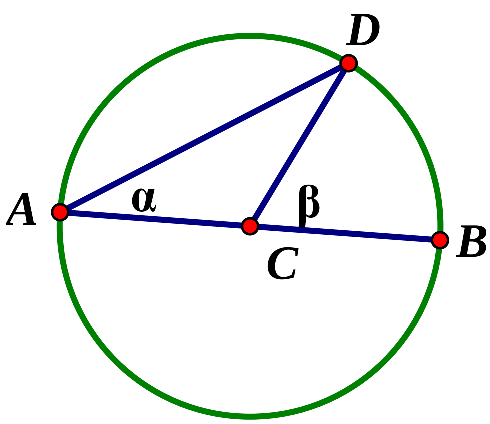
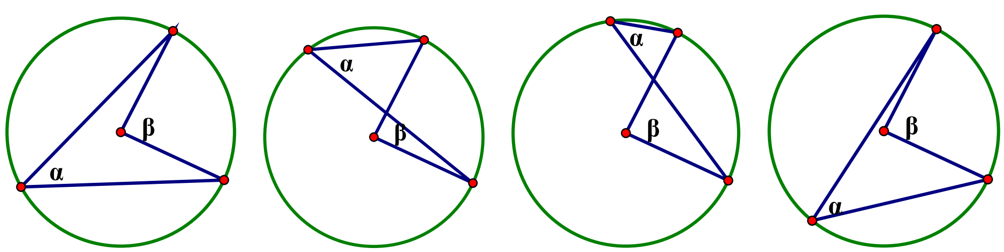
Some more circles to investigate. In the following problems, you may use:

- All of the theorems we have proved so far
- The Pythagorean theorem
- The converse of the Pythagorean theorem (ie. if $a^2 + b^2 = c^2$ for the sides of the triangle, then it is a right triangle.
- Relationships among similar triangles (eg. if two triangles have congruent angles, then the triangles are similar, and the ratios of corresponding sides are equal)
- The segment from the center of a circle to the midpoint of a chord is the perpendicular bisector of the chord.
- A tangent line to a circle is perpendicular to the radius at the point of tangency.

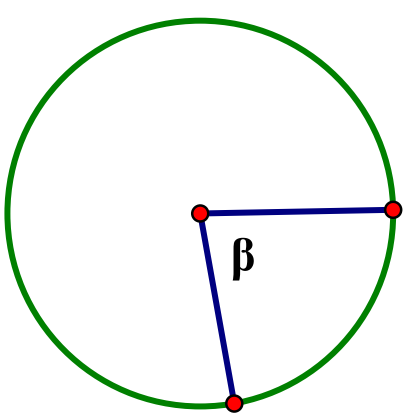
1. In this circle, $\overline{AB}$ is a diameter of the circle, and C is the center of the circle. There is a relationship between the inscribed angle α and the central angle β: $\beta = 2\alpha$. Prove the relationship (use theorems you know about the triangles shown.).



2. This relationship is true of any pair of central and inscribed angles. Here are some examples:



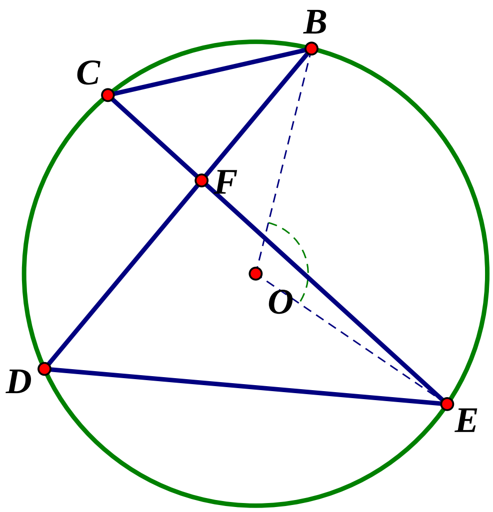
In this circle, draw in 3 different inscribed angles that all have the same measure as $\frac{1}{2}\beta$ in this circle:



3. In this diagram:
a. Find (and write) two angles that are congruent because they are both inscribed angles whose measure is half of $m\angle BOE$
b. Find (and write) two angles that are congruent because they are vertically opposite

c. Find (and write) two triangles that are similar

d. Write down a proportion that is true because of the similar triangles (a proportion is an equation that says one ratio is equal to another ratio)



4. The central/inscribed angle relationship $\beta = 2\alpha$ works even if the central angle lies on a straight line. What does that tell you about angle α triangle ΔABD?