Equilateral triangle: List the steps for constructing an equilateral triangle, and prove that the resulting construction is equilateral and equiangular.

Given: a segment $\overline{AB}$

Construction: Construct circle C_1 with center A and radius $\overline{AB}$

Construct circle C_2 with center B and radius $\overline{AB}$.

Let C be one of the intersection points of the two circles.

Construct segments $\overline{BC}$ and $\overline{AC}$

Claim: ΔABC is an equilateral triangle.

Proof: $\overline{BC}$ and $\overline{AC}$ both have the same length as $\overline{AB}$ because they are radii of circles with radius $\overline{AB}$, hence the triangle is equilateral.

Perpendicular line through a point on the line (one of many versions): List the steps for constructing a line perpendicular to a given point, and through a given point on the line, and prove that the resulting construction is a perpendicular line that passes through the given point.

Given: line l and point $P \in l$

Construction: Construct a circle C_0 with center P and radius $r > 0$

Let points A and B be the intersections of line l and circle C_0.

Construct circle C_1 with center A and radius $\overline{AB}$

Construct circle C_2 with center B and radius $\overline{AB}$.

Let C be one of the intersection points of the two circles.

Construct the line $\overleftrightarrow{CP}$

Claim: $\overleftrightarrow{CP}$ contains point P and is perpendicular to line l.

Proof: $\overleftrightarrow{CP}$ contains point P by construction.

$\overline{AC} \cong \overline{BC}$ because they are radii of circles with radius $\overline{AB}$

$\overline{AP} \cong \overline{BP}$ because they are radii of circle C_0

$\overline{CP} \cong \overline{CP}$ since it is the same segment.

So, by SSS, $\Delta ACP \cong \Delta BCP$

Hence, $\angle APC \cong \angle BPC$

$m\angle APC + m\angle BPC = 180°$ since the angles make a straight line, and so by substitution $2m\angle APC = 180°$

So we can conclude that $m\angle APC = 90°$ and $\overleftrightarrow{CP}$ is perpendicular to line l.

Congruent angle: List the steps for constructing an angle congruent to a given angle on a given side of a given ray, and prove that the resulting construction is congruent to the given angle.

Given: An angle $\angle BAC$ and a ray $\overrightarrow{DE}$ and a side of the line $\overleftrightarrow{DE}$

Construct: Construct a circle $C_{A,B}$ with center A and radius $\overline{AB}$

Let C' be the intersection of circle $C_{A,B}$ with the ray $\overrightarrow{AC}$

Construct circle $C_{D,AB}$ with center D and radius $\overline{AB}$

Let E' be the intersection of circle $C_{D,AB}$ and ray $\overrightarrow{DE}$

Construct a circle $C_{E',BC'}$ with center E' and radius $\overline{BC'}$

Let F be the intersection of $C_{E',BC'}$ with $C_{D,AB}$ on the given side of $\overleftrightarrow{DE}$

Construct ray $\overrightarrow{DF}$

Claim: $\angle EDF \cong \angle BAC$

Proof: $\overline{AB} \cong \overline{DE'}$ and $\overline{AC'} \cong \overline{DF}$ since they are all radii of circles with radius $\overline{AB}$
$\overline{E'F} \cong \overline{BC'}$ since they are radii of circles with radius $\overline{BC'}$
So, $\Delta BAC' \cong \Delta E'DF$ by SSS
Hence $\angle BAC = \angle BAC' \cong \angle E'DF = \angle EFD$ (CPCTC)

Parallel line:

Given: line l and point P not on the line.

Construct: Let m be a line through P that is perpendicular to line l (construction of perpendicular through a point not on the line)
Let n be a line through P that is perpendicular to line m (construction of a perpendicular through a point on the line)

Claim: n is parallel to l

Proof: Let $\angle a$ and $\angle b$ be alternate interior angles to line m between lines l and n (note that l and n do not intersect at line m because n contains point P and l does not).
By construction $m\angle a = 90° = m\angle b$ and hence $\angle a \cong \angle b$.
By theorem 19, l is parallel to n.

Perpendicular line through a point not on the line: List the steps for constructing a line perpendicular to a given point, and through a given point not on the line, and prove that the resulting construction is a perpendicular line that passes through the given point.

Given: a line l and a point P not on l.

Construct: Let Q be a point on the other side of l from P.

Construct circle $C_{P,Q}$ with center P and radius $\overline{PQ}$

Let A and B be the points of intersection of circle $C_{P,Q}$ with line l.

Construct circle C_1 with center A and radius $\overline{PQ}$

Construct circle C_2 with center B and radius $\overline{PQ}$.

Since A and B lie on circle $C_{P,Q}$, we know that $\overline{AP} \cong \overline{PQ} \cong \overline{BP}$ and hence P is on C_1 and C_2

Let D be the point of intersection of C_1 and C_2 on the opposite side of l from P.

Construct the line $\overleftrightarrow{DP}$

Claim: $\overleftrightarrow{DP}$ is perpendicular to l.

Proof: Let E be the intersection of $\overleftrightarrow{DP}$ and l.
$\overline{AP} \cong \overline{BP}$ and $\overline{AD} \cong \overline{BD}$ because they are all radii of circles with radius $\overline{PQ}$.
$\overline{PD} \cong \overline{PD}$ (same segment)
So $\Delta APD \cong \Delta BPD$ by SSS
And thus $\angle APE = \angle APD \cong \angle BPD = \angle BPE$ (CPCTC)
We also know that $\overline{PE} \cong \overline{PE}$ (same segment)
And together with the results above that $\overline{AP} \cong \overline{BP}$ and $\angle APE \cong \angle BPE$ we can conclude that $\Delta APE \cong \Delta BPE$
Thus $\angle AEP \cong \angle BEP$
Since $\angle AEP$ and $\angle BEP$ together make a straight line, $m\angle AEP + m\angle BEP = 180°$
By substitution
$2m\angle AEP = 180°$ and hence
$m\angle AEP = 90°$
Thus $\overleftrightarrow{DP}$ is perpendicular to l.