Additional Notes to the Teacher

Lesson 1

The RNP level 1 lessons support students' development of informal ordering strategies. Four informal ordering strategies have been identified: same numerator, same denominator, transitive, and residual. These strategies are not symbolic ones, but strategies based on students' mental representations for fractions. These mental representations are closely tied to the fraction circle model.

Same denominator: When comparing $\frac{4}{5}$ and $\frac{3}{5}$ students can conclude that $\frac{4}{5}$ is larger because when comparing parts of a whole that are the same size (in this case 5^{ths}) then 4 of those parts are bigger than 3 of them.

Same numerator: When comparing $\frac{4}{5}$ and $\frac{4}{6}$, students can conclude that $\frac{4}{5}$ is bigger because 5ths are larger than 6ths and four of the larger pieces will be bigger than 4 of the smaller pieces. Students initially come this understanding by comparing unit fractions.

Transitive: When students use benchmark of ½ and one they are using the transitive property. When comparing $\frac{6}{14}$ and $\frac{9}{16}$ students can conclude that $\frac{9}{16}$ is larger because $\frac{6}{14}$ is a little less than ½ and $\frac{9}{16}$ is a little more than ½.

Residual: When comparing fractions $\frac{2}{3}$ and $\frac{3}{4}$ students can decide on the relative size of each fraction by reflecting on the amount away from the whole. In this example, students can conclude that ¾ is larger because the amount away from a whole is less than the amount away from the whole for $\frac{2}{3}$. Notice that to use this strategy students rely on the same numerator strategy; they compare $\frac{1}{4}$ and $\frac{1}{3}$ to determine which of the original fractions have the largest amount away from one.

Students who do not have experiences with concrete models like fraction circles or students who may not have sufficient experiences with models to develop needed mental representations to judge the relative size of fractions using these informal strategies make consistent errors. On the next page we share with you examples of students' errors based on written test given to students after their RNP level 1 review lessons. We also share examples of correct thinking among this group of sixth graders. In all questions students were asked to circle the larger of the two fractions.

Misunderstandings

Students often focus on the denominator only after internalizing the relationship between the size of the denominator and the size of the fractional part. To understand what a fraction means, students need to coordinate the numerator and denominator - an idea the following students did not do.

Same numerator good here

$\frac{4}{9}$ (needs 5 pcs.)	$\frac{4}{15}$ (need 11 pcs.)	This is bigger because you need ~~less~~ to make a whole than the other fraction. R

??? think about size of missing pieces

$\frac{3}{4}$	$\frac{2}{3}$	Smaller denominator

residual good here

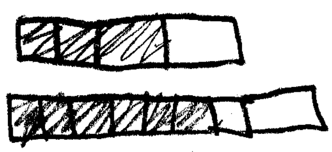
not looking at number of pieces

$\frac{3}{4}$	$\frac{2}{3}$	I got my answer by knowing that thirds are bigger than fourths

know size

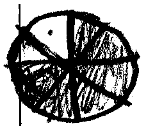
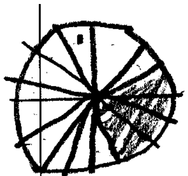
$\frac{6}{14}$	$\frac{9}{16}$	in Fractions 14 is biger than 16

An underlying assumption when ordering fractions is that the units for both fractions must be the same. Not realizing the unit needs to be the same is a common error as shown in this student's picture.

$\frac{3}{4}$	$\frac{6}{8}$	

not same whole

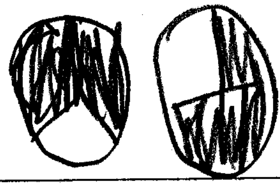
Whole number thinking also dominates students thinking when they first start working with fractions. This is shown in different ways. Without models students might say that $6/8 > 3/4$ because 6>3 and 8>4. But even after students use concrete models, their whole number thinking may still dominate. In the following examples, note that some students determine the larger fraction by deciding which fraction had the larger number of pieces. In other cases, students look at the difference between numerator and denominator to identify the larger fraction. In both instances, students have yet to focus on the relative size the fractional part being examined. These students need more time with concrete models to overcome their whole number thinking.

$\frac{3}{4}$	$\frac{6}{8}$	 has more peices.	know → numerator more pieces not thinking about how much of a whole didnt draw both
$\frac{4}{9}$	$\frac{4}{15}$	 has more peices.	didn't use: size of pieces (denom.)
$\frac{3}{4}$ $\neq$	$\frac{2}{3}$	There both equal because there both one fraction from a whole.	also need size of piece missin
$\frac{3}{4}$	$\frac{6}{8}$	it only need 1 more to be hale	

Understandings

But with enough experiences with concrete models, students do overcome these misunderstandings. Below find student examples for the transitive and residual strategies:

$\frac{6}{14}$	$\frac{9}{16}$	$\frac{6}{14}$ is a little less then $\frac{1}{2}$ and $\frac{9}{16}$ is a little more.
$\frac{6}{14}$	$\frac{9}{16}$	$\frac{9}{16}$ is bigger because its over a half and $\frac{6}{14}$ is less than a half.

$\frac{3}{4}$	$\frac{2}{3}$	 There is a bigger space left on $2/3$
$\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{3}$ needs a bigger perce than $\frac{3}{4}$ sence the donomintator is smallar.

When you teach lesson 3 you will notice students using these informal ordering strategies along with other benchmarks to estimate fraction and subtraction problems effectively.