**Problem solving in the Common Core State Standards for Mathematics**

**1. Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**“How to Solve It” by George Polya**

**Problem solving steps**

1. Understand the problem

2. Make a plan for how to solve it

3. Carry out your plan

4. Look back and evaluate your progress or your solution

**Questions to help you understand the problem**

1. What is the unknown? What is the problem asking for?

Try to restate the problem in your own words

Is there a diagram or picture or table that would help you represent the problem?

2. What are the conditions or constraints that a solution must have?

Do you understand all of the conditions, or do you need to ask a question?

3. What is the data or information that is given?

Is there enough information given, or do you need to ask a question or make an assumption? (If you make an assumption in solving a problem, make sure that you explain your assumption clearly)

**Some strategies to help you make a plan**

1. Try some examples. Make a guess, and check it to find out how it works. Organize your guesses, so you can learn from them.

2. Look for patterns in the problem or in your examples.

3. Draw a picture or a diagram, or try to act out the problem using a picture or a diagram or manipulatives.

4. Look for a simpler or more familiar problem that is similar to the problem you are trying to solve and solve it first, and try to apply the same strategy to the more difficult problem.

5. Try working backward

**Problems to think about**

1. The school music problem:

A school is taking three music organizations to a festival.

30 students sing in the choir

40 students play in the orchestra

50 students march in the band

15 students belong to the choir and the orchestra

10 students belong to the orchestra and the band

14 students belong to the choir and the band

5 students belong to all three groups

How many students will need seats on the bus?

2. The box problems:

a. A jeweler has a packing box that is 1 ft3 (1 ft. x 1 ft. x 1 ft.). In the packing boxes are boxes holding rings that are each 1 in3 (1 in. x 1 in. x 1 in.). How many ring boxes can fit in the packing box?

b. I have a packing box that is 1 ft3. I have a lot of books. Each book is no larger than 1 in. x 6 in. x 4 in.

i. What is the volume of a book of the maximum size?

ii. How many books (at least) will fit in my box?

c. I have a packing box that is 3 ft3. I have boxes of candy that are each 12 in3. How many boxes of candy will fit in my packing box?