# Fractions, Ratios and the Number system:

Models: We’re going to use three of the most common models or visualizations for fractions: geometric fractions of a circle and a rectangle, and number line fractions of a length. Each of these models has advantages and disadvantages for learning about fractions. In grade 2, students are introduced to fractions using geometric shapes (such as circles and rectangles) and in grade 3, students are introduced to fractions on a number line.

Most of the visualizations we draw for fractions, are built on an area model of a fraction: the relationship between the area of the part and the area of the whole represents the fraction.

In a **circle** model, the circle represents the whole. The circle is subdivided using radii (segments from the center of the circle to the perimeter), that cut the circle into sectors with equal area. These sectors represent the fractional amount when compared to a whole that is represented by the entire circle.

A circle model of fractions has the advantage that our brains are pretty good at recognizing and estimating angle sizes, so the visualization of the size of a third or a fourth or a fifth is something we’re pretty good at remembering and estimating. Practice your estimation strategies on these circles by subdividing them:

   

 thirds fourths fifths sixths

In a **rectangular** model, a rectangle or square (a square is just a special kind of rectangle) represents the whole. The whole is then subdivided using horizontal and vertical lines to make equal sized parts. Rectangles and squares can also be divided into halves or fourths using diagonal lines, but those aren’t going to give us the most useful properties that rectangular models have, so we’re going to just use subdividing lines that are parallel to the sides.

Rectangular models get really useful when you are making subdivisions in both directions at once. For instance, you could subdivide these squares square into sixths in two ways: either by making all of the subdivisions parallel to the same side, or by making some dividing lines vertical and others horizontal.

|  |  |  |  |
| --- | --- | --- | --- |
| Show sixths using only vertical lines | Show sixths using some vertical and some horizontal lines | In the pictures of sixths you just drew, can you see thirds in the same picture? Can you see halves? |  How would you draw 20ths on a rectangular diagram? |

The **number line** model of fractions shows a fraction as a length. The length of a whole unit is marked and labelled as 1 (it is a marked length not a separate shape like a circle or a square). Fraction bars and tape diagrams (which are also called bar diagrams) are variations on the number line model.

Number line fractions are particularly good for representing improper fractions, for comparing fractions to whole numbers, and for thinking of fractions as a kind of number rather than a kind of shape.

On this number line, show the fractions 2/3, and 5/2:



|  |
| --- |
| **CCSS:** In grade 2 children work with simple fractions of geometric shapes:[CCSS.Math.Content.2.G.A.3](http://www.corestandards.org/Math/Content/2/G/A/3) Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths…In grade 3, the number line model for fractions is introduced, and is a topic of particular focus:[CCSS.Math.Content.3.NF.A.2](http://www.corestandards.org/Math/Content/3/NF/A/2) Understand a fraction as a number on the number line; represent fractions on a number line diagram. |

## Building on Unit Fractions: Understanding fractions as numbers (grades 3-5)

The most important idea for basic fraction knowledge in the common core standards is how to present fractions in terms of unit fractions, as described below:

A **unit fraction** is a fraction whose numerator is 1. To show the unit fraction 1/b in any of our models, we start with a representation of 1 whole, and divide it into b equal parts. Each of those parts represents the fraction 1/b. This is always the way that these fractions are defined and presented, so only the name *unit fraction* should be new to you.

Fractions with a numerator greater than 1 are represented and explained as a sum of unit fractions, so 2/3 is 2 units of size 1/3, and 9/4 is 9 units of size ¼. This is slightly different from the most common way of presenting such fractions. The most common way of explaining 2/3 is to say that it is two out of 3 equal parts of a whole. That’s a fine explanation for fractions that are less than 1 (proper fractions), but it leads to misunderstandings when children encounter fractions that are greater than 1 (improper fractions). This is the most fundamental change suggested for teaching and understanding fractions in the Common Core Standards compared to the way things have usually been done in the past, and it is a topic of emphasis in grade 3.

Practice: Using a number line and a circle model, show the steps (using unit fractions) to represent the fraction 7/4. Write a sentence explaining each step.

* The first step is to divide the whole unit into equal parts to show the size of the unit fraction (1/4)
* The second step is to draw out the right number of unit fractions to make the fraction you want to show (7/4)

|  |  |
| --- | --- |
|  |  |

|  |  |
| --- | --- |
|  Unit fraction   |  |
| Repeated units: |  |

|  |
| --- |
| CCSS: Understanding fractions as repeated unit fractions is an explicit standard at third grade, and is again reiterated at fourth grade:[CCSS.Math.Content.3.NF.A.1](http://www.corestandards.org/Math/Content/3/NF/A/1) Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.[CCSS.Math.Content.4.NF.B.3](http://www.corestandards.org/Math/Content/4/NF/B/3) Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. |

##

## Fraction Equivalence and Comparing Fractions:

The teaching of fraction equivalence and fraction comparison is divided into two stages: a visual reasoning stage and a computational reasoning stage.

* In the *visual reasoning* stage, the main concepts are that the sizes of fractions can be compared using manipulatives and diagrams and by reasoning directly from the size of unit fractions. (grades 2-3 in CCSS)
* In the *computational* stage, students learn to generate equivalent fractions using multiplication and fractions are compared by finding a common denominator (usually) or a common numerator (occasionally). (grade 4 and up)

A **visual** approach to **equivalent fractions** is to say that *two fractions are equivalent if they show the same part of the whole*. In an area model (circles or rectangles), that means that the fractions cover the same area. In a number line (length) model, that means the fractions are at the same point on the number line or show the same length. Reasoning about this equivalence is often done by comparing physical models. It’s easy to make mistakes with pictures and manipulatives if the unit fractions are very small, but experience with pictures and manipulatives helps children develop visualization and estimation skills.

Another detail to notice is that the notation for writing a whole number as a fraction (2=2/1) is something that is now introduced in third grade along with putting fractions on a number line.

 

A **visual** approach to **comparing fractions** is to show that one fraction is larger than another if it shows a larger part of the same whole. In an area model, the area is larger, and in a length model the length is longer. The concept that *the size of the whole must be the same in order to compare fractions* is a key goal of instruction at this level. For example, to compare two fractions on number lines, the length of the whole unit 1 must be the same on both number lines:







The size of the rectangle that shows 1

must be the same for both fractions

in a rectangular model:

|  |
| --- |
| **CCSS grade 3**[CCSS.Math.Content.3.NF.A.3](http://www.corestandards.org/Math/Content/3/NF/A/3) Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.[CCSS.Math.Content.3.NF.A.3a](http://www.corestandards.org/Math/Content/NF/A/3/a) Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.[CCSS.Math.Content.3.NF.A.3b](http://www.corestandards.org/Math/Content/NF/A/3/b) Recognize and generate simple equivalent fractions… . Explain why the fractions are equivalent, e.g., by using a visual fraction model.[CCSS.Math.Content.3.NF.A.3c](http://www.corestandards.org/Math/Content/NF/A/3/c) Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.[CCSS.Math.Content.3.NF.A.3d](http://www.corestandards.org/Math/Content/NF/A/3/d) Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. |

**Reasoning** **with unit fractions** can explain some comparisons without relying on a picture and also without computing common denominators. This kind of reasoning builds a good foundation for future fraction learning. The two most important examples of this are comparisons when either the denominators are already the same or the numerators are already the same in the two fractions.

* If the denominators are the same, then the fractions are composed of *unit fractions of the same size*, so the fraction with more units is larger.
* If the numerators are the same, then the fractions are composed of the same number of unit fractions, so we should compare the size of the unit fractions. Remember that unit fractions are made by dividing a whole into a given number of equal sized pieces. *If the whole is divided into more pieces, each unit fraction must be smaller*. The fraction with the larger denominator is composed of smaller unit fractions, so its size must be smaller.

Explaining comparisons using unit fractions is probably new to you. Practice with these pairs of fractions:

|  |  |  |
| --- | --- | --- |
| Explain:* Are the unit fractions the same?
* If the unit fractions are not the same, which is larger? Why?
* Are there the same number of unit fractions?
* How does the number of unit fractions and the size of unit fractions tell you which fraction shows the larger amount?
 | Which is larger, 7/15 or 8/15? | Which is larger, 5/8 or 5/9? |