Test practice problems for TED 323

Information given on the test/on the board:
CGI problem types are:

<table>
<thead>
<tr>
<th>Join, Result Unknown (JRU)</th>
<th>Join, Change Unknown (JCU)</th>
<th>Join, Start Unknown (JSU)</th>
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</thead>
<tbody>
<tr>
<td>Separate, Result Unknown (SRU)</td>
<td>Separate, Change Unknown (SCU)</td>
<td>Separate, Start Unknown (SSU)</td>
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<tr>
<td>Part Part Whole, Whole Unknown (PPW-WU)</td>
<td>Part-Part-Whole, Part Unknown (PPW-PU)</td>
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<tr>
<td>Compare, Difference Unknown (CDU)</td>
<td>Compare, Compared Quantity Unknown (CQU)</td>
<td>Compare, Referent Unknown (CRU)</td>
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Sample test questions:

1. For each pair of problems on this page, circle the more difficult problem and write a sentence explaining why it is more difficult (for a child at the direct modeling stage of solving addition and subtraction problems).

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>Problem 2</th>
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<tbody>
<tr>
<td>Jeff has 3 puzzles. Todd has 4 puzzles. How many puzzles do they have all together? PPW-WU</td>
<td>Jeremy made 5 paper airplanes. Later he made 2 more paper airplanes. How many paper airplanes did he make in all? JRU easier</td>
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<td>harder because PPW-WU is harder than JRU. Some children have a hard time seeing the two sets as put together in a bigger set.</td>
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<td>John had 6 glow in the dark bugs. When he cleaned his room, he found some more glow in the dark bugs, and then he had 10 glow in the dark bugs. How many glow in the dark bugs did he find? JCU</td>
<td>Ben built 7 block towers. How many more does he have to build to have 11 block towers? JCU</td>
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<td>JCU in the past are harder.</td>
<td>How many more JCUs are easier</td>
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<td>Ben has 3 small toy cars. He has 9 more large toy cars than small toy cars. How many large toy cars does he have? CQU</td>
<td>Clara has 11 Barbies. She has 5 more Barbies than Anne. How many Barbies does Anne have? CRU</td>
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<td>CRU is harder than CQU. With CQU “9 more” tells us to do 3+9. In CRU “5 more” but we have to work back and do 11-5.</td>
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<td>There are 7 children running in the race. 3 of the children are boys. How many of the children are girls? PPW-PU</td>
<td>Yesterday Gus made some origami animals. Today, he made 2 more origami animals. In all, he made 6 origami animals. How many origami animals did he make yesterday? JSU</td>
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<td>JSU is harder because start unknown problems are hard to direct model.</td>
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<td>Kyle has 7 stuffed toy animals and 8 hard plastic toy animals. How many more hard plastic toy animals than stuffed toy animals does Kyle have? CDU</td>
<td>Michelle had some toy animals. She gave 10 toy animals to Jane. Now she has 6 left. How many toy animals did Michelle have to begin with? SSU SSU is harder than CDU (start unknown problems are hard to direct model)</td>
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<tr>
<td>Briana had 9 Silly Bandz. She gave 3 Silly Bandz to Laura. How many Silly Bandz does Briana have left? SRU</td>
<td>2 of Leah’s crayons got lost. She started with 9 crayons. How many crayons does she have now? SRU Harder because information is out of order.</td>
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<tr>
<td>There are 12 balloons in the room. 7 of the balloons are mylar and the rest are latex. How many of the balloons are latex? PPW-PU</td>
<td>There are 14 ounces of mixed juice in the pitcher. 8 ounces are apple juice, and the rest are grape juice. How many ounces of grape juice are in the pitcher? PPW-PU This one is harder because the numbers are representing a measured amount (liquid) and not something discrete that you can easily count one by one.</td>
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2. Explain the difference between each of these problem types:

a. JRU and PPW-WU: JRU has a change over time: there is one group, and some is added to that group. PPW-WU, there are two groups that are thought of as one group.
b. CQU and CRU: In CRU the referent (set following “than”) is unknown, and in the CQU, the referent is known and the other set that is being compared is unknown. In CQU problem the comparison (more than/less than) tells you how to get the answer. In CRU, what you need to do is backwards from the comparison (more/less)
c. PPW-WU and CDU:
In PPW the two sets are combined (question about sum)
In CDU the two sets are compared. (question about difference)
d. JSU and SSU:
Join—the result is bigger than the start
Separate, the result is less than the start

3. Do children first figure out problems where there is a change over time, or problems where there is no change over time?
First figure out change over time.

4. Which problem types have a change over time?
Join and separate

5. Draw out or explain how a child would direct model (an example of) each of these problem types:
JRU (using the joining all strategy). JCU (using joining to). SRU (using separating from). CDU (using comparing)

JRU: S has 5 apples, she gets 3 more. How many now?

Put out 5 counters. Add 3 counters to the pile. Count all for answer.

JCU (join to): S has 5 apples. How many more does she need to get to have 8 apples?

Put out 5 counters. Put out more counters (in a separate group) and count on until you get to 8. Count how many in the added group:

SRU: (separating from): S has 8 apples. She eats 3 apples. How many apples does she have left?

Put out 8 counters. Move 3 counters away (separate them from the pile). Count how many are left.

CDU (comparing): S has 8 apples. J has 5 apples. How many more apples does S have than J?

Put out 8 counters in one pile and 5 in another. Match up counters next to each other, and count how many are left over.

6. Write a problem for a given problem type (eg. JRU)
7. Is it a good idea to tell children to add when they see the word "more" and subtract when they see the word "fewer" or "less"? Why or why not?

Not a good idea, because it doesn’t always work, especially with compare problems:

John has 5, and Sue has 7. How many more does Sue have than John? (subtract)

John has 5. Sue has 2 more than John. How many does Sue have? (add)

Sue has 7. Sue has 2 more than John. How many does John have? (subtract)
(problems with the word more are sometimes solved by adding, and sometimes by subtracting).

8. Explain how students might solve a word problem (eg. Sarah caught 5 fish. How many more does she have to catch to have 8 fish?)
   a. By direct modeling

   (See joining to strategy above)

   b. Using a counting strategy.

      count 5…6, 7, 8, putting up a finger with the counted on amounts (6, 7, 8). The answer is 3
      (number of counts or fingers)

9. Give at least 3 examples of basic facts for which…
   a. counting on is an efficient strategy—addition
      8+2, 9+3, 6+1, 2+6
      (all of the examples should have +1, +2 or +3)

   b. counting back (by subtrahend) is an efficient strategy
      9-2, 7-1, 8-2 (8-3)
      (all of the examples should have -1, -2 or maybe -3)
   c. counting up to (for subtraction) is an efficient strategy
      9-7, 8-6, 6-5 (9-6)
      (all of the answers should have a difference—the answer—of 1, 2 or 3)

10. Which of these are considered basic facts?

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<tbody>
<tr>
<td>8+7</td>
<td>9-3</td>
<td>13+5</td>
<td>13-5</td>
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<tr>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>17-4</td>
<td>N</td>
<td></td>
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</tbody>
</table>

11. A child solves 5+3 by making 5 tally marks and 3 tally marks and counting all of them. Is this considered a counting strategy or a direct modeling strategy?

   Direct modeling (because all of the numbers are shown in the drawing—the problem is completely acted out)
12. How is counting on from first different from counting on from higher?  
JRU, \(3 + 8\)   3…4, 5, 6, 7, 8, 9, 10, 11 count on from first.  8…9, 10, 11 counting on from higher

13. What CGI type is most strongly associated with each of these counting strategies?  
a. counting on JRU  
b. counting back SRU  
c. counting up to (for subtraction) JCU

14. What are the two different variations on counting back that a child might use to solve 11-2?  
11…10, \(\boxed{9}\) (put up a finger for 10 and a finger for 9)  ---  11, 10…\(\boxed{9}\) (put up a finger for 11 and a finger for 10).

15. Why do we want children to learn to use "counting up to" to solve subtraction problems?  
Counting up to is related to thinking of 11-8 as meaning: how much bigger is 11 than 8? and How many more than 8 is 11?  
It shows more closely than a counting back strategy that subtraction is related to addition.  
Also accurately counting up is easier than counting back.

16. See below...

17. Write a good word problem for introducing  
a. counting on (+) It should be a JRU, where the first addend is fairly large, and the second addend is 1 or 2:  ___+1,   ___+2  
b. counting back (-) It should be an SRU problem, where you are taking away only 1 or 2:  ___-1,   ___-2.  The start amount should be not too small (bigger than 5)  
c. counting up to (-):  It could be any of the problem types: JCU, PPW-PU, CDU, and it should be a problem where you would only need to count up 1 or maybe 2, such as  7-6,  8-7,  9-7.  
Both the minuend and the subtrahend should be not too small (bigger than 5)

16. Lessons for counting on:

Word problem based lesson  
The teacher poses a word problem to a group of children, and asks the children to solve it:  
Matthew had 12 paper airplanes. He made 2 more paper airplanes. How many paper airplanes does he have now?  
The teacher waits while children solve the problem, and then invites 2-3 children to share how they solved the problem.

The teacher highlights a counting on solution:  
I'd like you to think about how Ellen solved the problem, she started with 12 and then counted the two more paper airplanes: 13 (raise a finger) 14 (raise a second finger). So she there were 14 in all.
The teacher poses a similar problem:
I have another problem for you to think about, and I'd like you to see if you can solve it Ellen's way:
Janet had 9 crackers. Her mom gave her 2 more crackers. How many crackers does Janet have now?

The teacher waits while children solve this problem, and then invites children to share a counting on solution:
Who can show how to solve this problem by counting on the way Ellen did?

Game based lesson:
The teacher has some counters (cubes or bear counters) and somewhere to hide some of them (a cup).
The teacher puts 7 counters in the cup all at once, and tells the children that there are 7 counters in the cup:
“My counters are going to be bears, and the cup is a cave. 7 bears went into the cave. Here are 2 more bears that are going to go in the cave. I want to know how many bears will be in the cave when these two go in?”

The teacher waits for the children to think, and then asks 2-3 children to share their solution.
The teacher highlights a counting on solution:
“Did you hear what Jack said? He started with the 7 that were in the cave, and then he counted the two that were out of the cave so he counted 7.... 8 [drop a bear in the cup] 9 [drop a bear in the cup].”

If the children want to check the answer, the teacher will invite someone up to take out the bears and count them.
The teacher asks another bears question:
“OK, I’ve got another bears question for you. This time there were 8 bears in the cave and 3 more came. Can you figure out how many will be in the cave when they all go in?” [put 8 counters in the cup and leave 3 out where they are easy to see]
The teacher calls on a child to share a counting on solution

Lessons for counting back
Word problem based lesson
The teacher poses a word problem to a group of children, and asks the children to solve it:
Matthew had 11 animal crackers. He ate 2 of them. How many animal crackers does he have left?

The teacher waits while children solve the problem, and then invites 2-3 children to share how they solved the problem.

The teacher highlights a counting back solution:
I'd like you to think about how Ellen solved the problem, she started with 11 and then counted back to show the ones he ate: He had 11 to start and he ate a cracker and had 10 (raise a finger), and he ate a cracker and had 9 (raise a second finger). So she there were left.

The teacher poses a similar problem:
I have another problem for you to think about, and I'd like you to see if you can solve it Ellen's way:
The balloon man had 9 balloons. He sold 2 of them. How many balloons does he have now?

The teacher waits while children solve this problem, and then invites children to share a counting on solution:
Who can show how to solve this problem by counting back the way Ellen did?

Game based lesson:
The teacher has some counters (cubes or bear counters) and somewhere to hide some of them (a cup).
The teacher puts 8 counters in the cup all at once, and tells the children that there are 8 counters in the cup:
“My counters are going to be bears, and the cup is a cave. There are 8 bears went in the cave right now. 2 of the bears come out of the cave (take out 2 counters one at a time and put them next to the cup). I want to know how many bears will are in the cave now?”

The teacher waits for the children to think, and then asks 2-3 children to share their solution. The teacher highlights a counting back solution:
“Did you hear what Jack said? He started with the 8 that were in the cave [put all the bears back in], and then he counted back for the two that were came out of the cave so he counted 8.... 7 [take a bear out] 6 [take a bear out].”

Show the process again, and have all of the children count back together. If the children want to check the answer, the teacher will invite someone up to take out the bears and count them.

The teacher asks another bears question:
“OK, I’ve got another bears question for you. This time there were 10 bears in the cave and 2 came out. Can you figure out how many are in the cave now?” [put 10 counters in the cup and take 2 out one at a time]

The teacher calls on a child to share a counting back solution

Lessons for counting up to for subtraction

Word problem based lesson
The teacher poses a word problem to a group of children, and asks the children to solve it:

Matthew has 9 Lego people. How many more does he need to get to have 11 Lego people?

The teacher waits while children solve the problem, and then invites 2-3 children to share how they solved the problem.

The teacher highlights a counting up solution:

I'd like you to think about how Ellen solved the problem, she started with 9 and then counted how many more Matthew would need: if he got another, he would have 10 (raise a finger), and if he got another, he would have 1 (raise a second finger). So he needs 2 more.

The teacher poses a similar problem:

I have another problem for you to think about, and I'd like you to see if you can solve it Ellen's way:

Sean has 7 green blocks. He also has some red blocks. In all he has 9 blocks. How many red blocks does he have?

The teacher waits while children solve this problem, and then invites children to share a counting on solution:

Who can show how to solve this problem by counting up the way Ellen did?

Game based lesson:

The teacher has some counters (cubes or bear counters) and somewhere to hide some of them (a cup).

The teacher puts 8 counters in the cup all at once, and tells the children that there are 8 counters in the cup:
“My counters are going to be bears, and the cup is a cave. There are 8 bears went in the cave right now. 7 of the bears come out of the cave (take out 7 counters out all at once and put them next to the cup). I want to know how many bears are in the cave now?”

*The teacher waits for the children to think, and then asks 2-3 children to share their solution. The teacher highlights a counting up solution:*

“Did you hear what Jack said? He started with the 7 that came out of the cave, and then he counted one more [hold up a finger] and got to 8, so he knew there was 1 bear left in the cave.”

Take the last bear out of the cave and count up again, 7 [point to the group of 7], 8 [point to the bear that was in the cup].

*The teacher asks another bears question:*

“OK, I’ve got another bears question for you. The 8 bears went in the cave, and this time, 6 came out. How many do you think are left in the cave?” [put 8 counters in the cup and take 6 out all at once]. I wonder if we can solve this one if we start counting with these 6 bears?

*The teacher calls on a child to share a counting up solution*