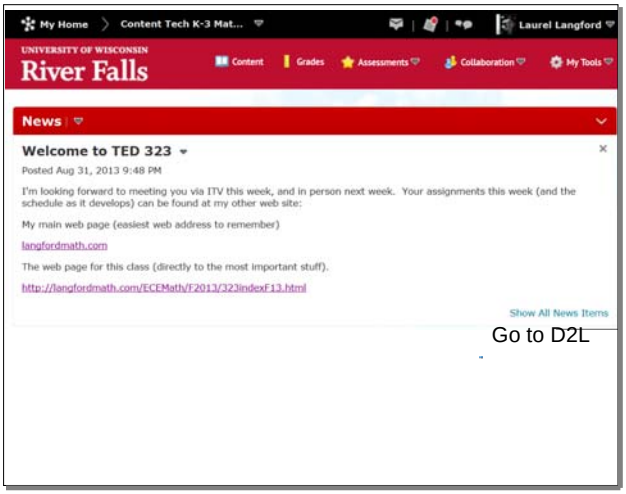
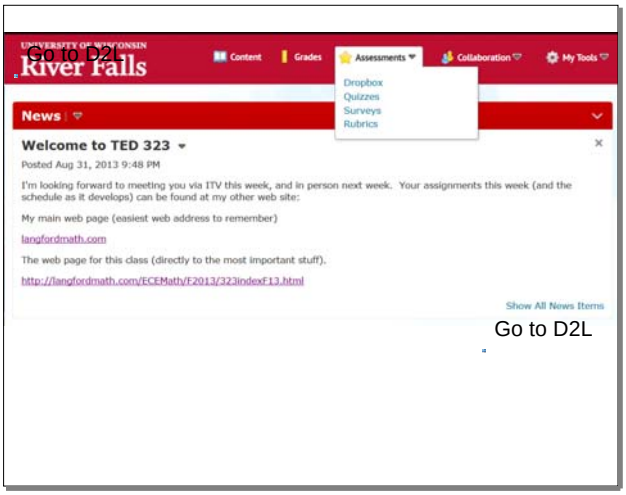
TED 323:
Math Content and Teaching Techniques K-3



Go to D2L



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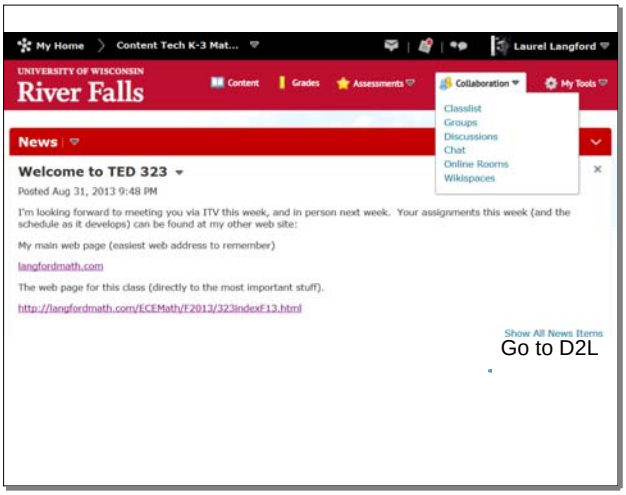

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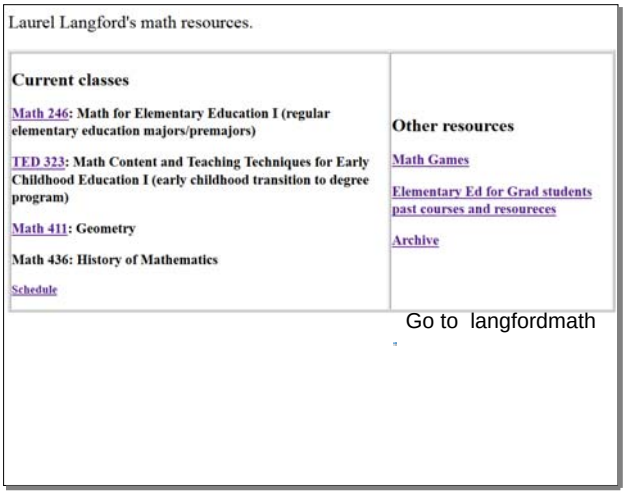

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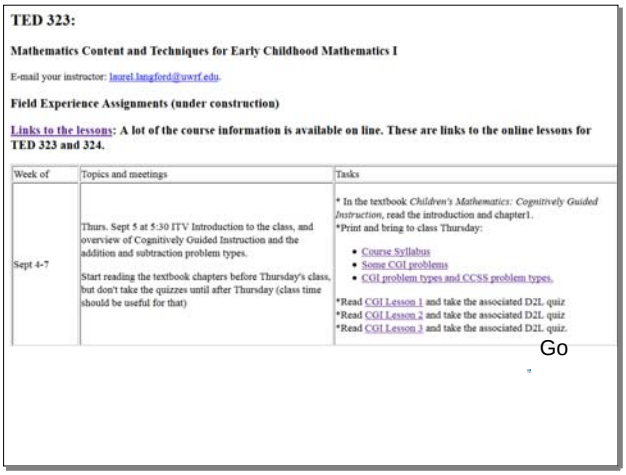

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CGI Word Problems and Strategies

Addition and Subtraction Strategies

Base 10 work and multidigit addition and subtraction

Multiplication and division

Algebraic Thinking

Geometry

Fractions

Go

Cognitively Guided Instruction:

Understanding how children think about addition, subtraction, multiplication and division, including...

- What contexts (word problems and visualization strategies) make it easier for children to visualize mathematical operations
- What stages children go through as they learn how to perform mathematical operations.
- How to write problems that support childrens mathematical learning
- What types of problems might be stumbling blocks for students at different levels

Addition and Subtraction:

In Children's Mathematics: Cognitively Guided Instruction, read the introduction, chapter 1, chapter 2, and the section Direct Modeling Strategies in chapter 3 (up to page 18).

Lesson 0: Introduction (no quiz)

Lesson 1: Two ways of looking at addition (JRU and PPW-WU) (D2L quiz)

Lesson 2: Addition and subtraction as actions (JRU and SRU, including direct modeling) (D2L quiz)

Lesson 3: Other problem types with distinct direct modeling strategies (JCU, SRU and CDU) (D2L quiz)

Lesson 4: More about part-part-whole (direct modeling by association: PPW-WU and PPW-PU)

Extra practice: Identify problems by their problem type and answer questions about direct modeling. (D2L quiz)

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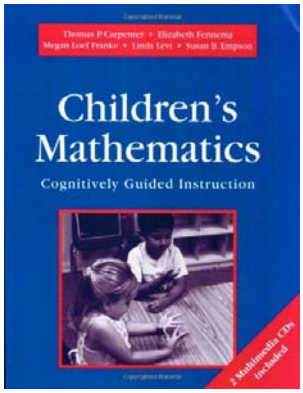
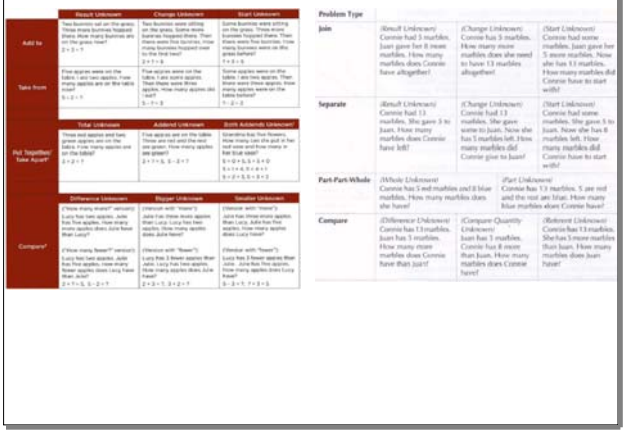


Research results about how children learn, solve and think about addition, subtraction, multiplication and division using word problem contexts.

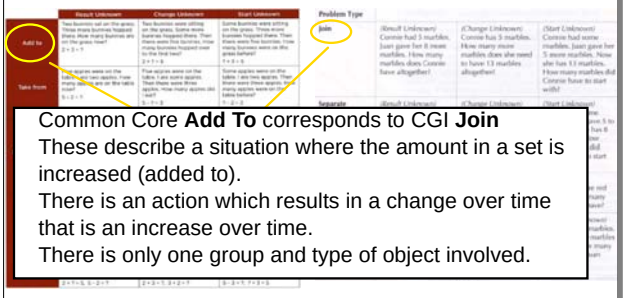
Common Core

CGI



Common Core

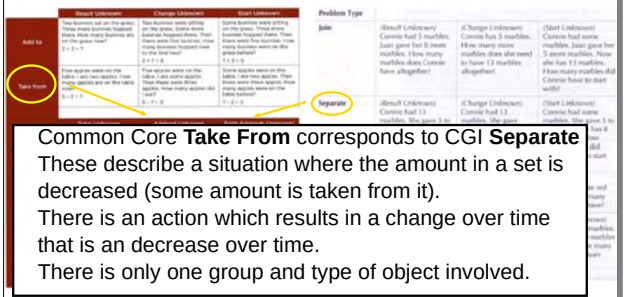
CGI



Common Core **Add To** corresponds to CGI **Join**
These describe a situation where the amount in a set is increased (added to).
There is an action which results in a change over time that is an increase over time.
There is only one group and type of object involved.

Common Core

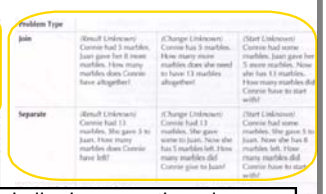
CGI



Common Core **Take From** corresponds to CGI **Separate**
These describe a situation where the amount in a set is decreased (some amount is taken from it).
There is an action which results in a change over time that is an decrease over time.
There is only one group and type of object involved.

Common Core

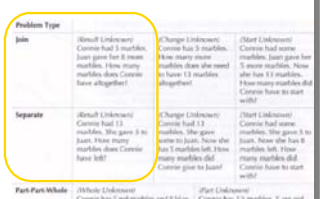
CGI



These first two groups are similar because there is an action which results in a change over time.
There is only one group and type of object involved in these problems.

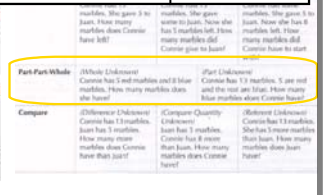
Common Core

CGI



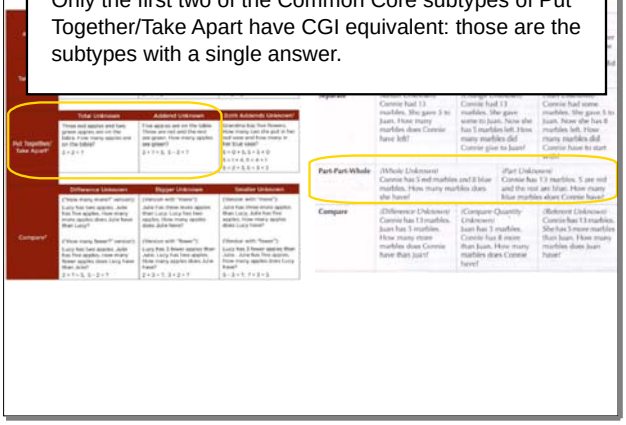
The result unknown versions of these problems are the easiest types of problems for children to understand and act out
The result is the end result after the action.

The CGI problem type **Part-Part-Whole** corresponds to the Common Core problem type(s) **Put Together/Take Apart**.
In this problem type a larger set is made of two smaller parts, and the sizes of the part and deduced from the part-whole relationship.
Typically there is no action involved in these problems.



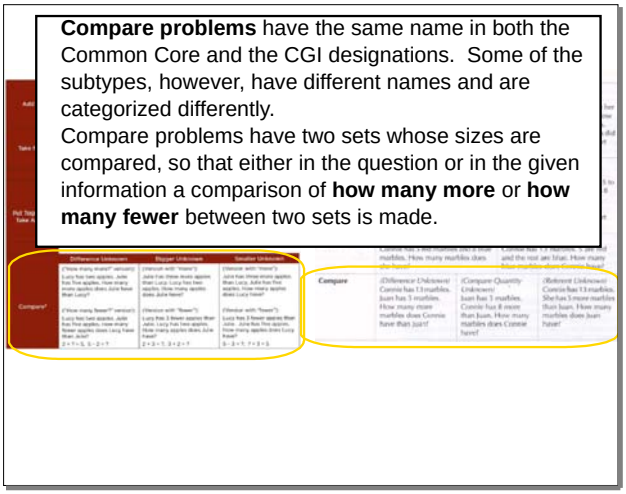
Common Core CGI

Only the first two of the Common Core subtypes of Put Together/Take Apart have CGI equivalent: those are the subtypes with a single answer.

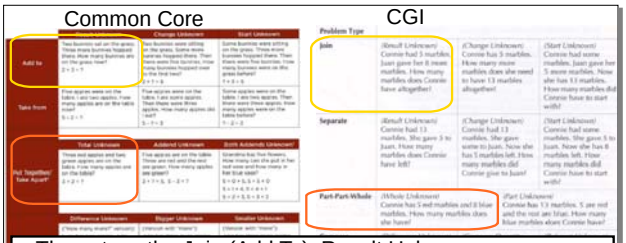


Compare problems have the same name in both the Common Core and the CGI designations. Some of the subtypes, however, have different names and are categorized differently.

Compare problems have two sets whose sizes are compared, so that either in the question or in the given information a comparison of **how many more** or **how many fewer** between two sets is made.



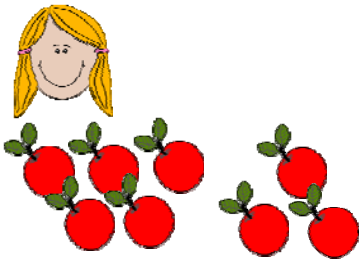
Common Core CGI



These two: the Join (Add To), Result Unknown and the Part-Part Whole, Whole Unknown (Put Together) are the trickiest to keep straight

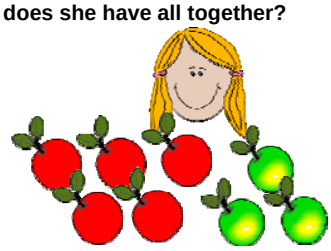
Join, Result Unknown, has an action:

Carol had 5 apples. She got 3 more apples. How many apples does she have now?

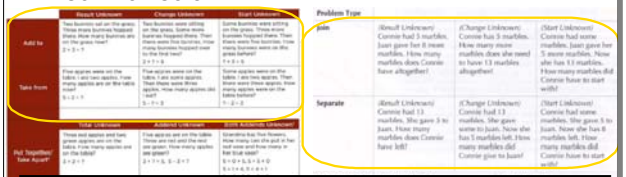


Part Part Whole, Whole Unknown
has subsets and does not have an action

Carol has 5 red apples and 3 green apples. How many apples does she have all together?



Common Core CGI

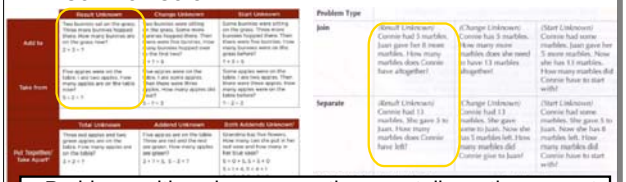


Join = Add to problems have an **increase over time**. The problems have an action that increases the amount originally had.
Separate = Take from problems have a **decrease over time**. The problems have an action that decreases the original amount.

CGI Practice Problems

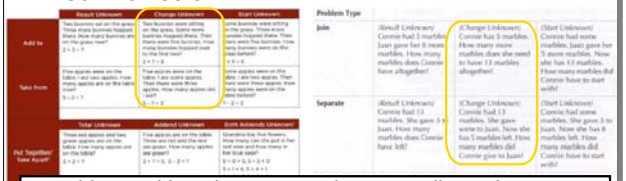
1. Gina had 13 glow sticks. She used up some of her glow sticks. Now she has 5 glow sticks left. How many glow sticks did she use up?	**11.** Kyle had some dinosaurs. He gave 5 dinosaurs to David. Now he has 4 left. How many dinosaurs did Kyle have to begin with?
2. There are 12 children sitting in the classroom. 5 of the children are girls. How many of the children are boys?	**12.** Gus had some grapes. He ate 3 grapes. Now he has 9 grapes left. How many grapes did he have before he ate any of them?
3. Gus has 2 crystal rocks and 7 polished rocks. How many more polished rocks than crystal rocks does Gus have?	**13.** Gus drew 3 pictures. How many more does he have to draw to have 6 pictures?
4. David has 1 new crayon. He has 7 more old crayons than new crayons. How many old crayons does he have?	**14.** Connor has 2 small toy cars and 9 large toy cars. How many toy cars does he have in all?
5. Blake has 3 glow in the dark toy bugs and 7 colored toy bugs. How many toy bugs does he have in all?	**15.** Donna has 6 sweet crackers. She has 1 fewer sweet crackers than salty crackers. How many salty crackers does she have?
6. Peter made 4 paper airplanes. Later he made 3 more paper airplanes. How many paper airplanes did he make in all?	**16.** Jane made 7 origami animals. Later she made 4 more origami animals. How many origami animals did she make in all?
7. Connor had some Bakugans. For his birthday, he got 4 more Bakugans. Now he has 11 Bakugans. How many Bakugans did he have before his birthday?	**17.** Donna had 14 carrot sticks. She ate 4 of her carrot sticks. How many carrot sticks did she have left?
8. Jill has 13 bouncy balls. She has 6 more bouncy balls than Nathan. How many bouncy balls does Nathan have?	**18.** Yesterday Barbra wrote some poems. Today, she wrote 5 more poems. In all, she wrote 6 poems. How many poems did she write yesterday?
9. Diana has 17 Barbies. Sandy has 9 fewer Barbies than Diana. How many Barbies does Sandy have?	**19.** Peter had 9 video games. He gave some video games to Sandy. Now he has 8 video games left. How many video games did he give to Sandy?
10. Blake had 8 bouncy balls. When he cleaned his room, he found some more bouncy balls, and then he had 11 bouncy balls. How many bouncy balls did he find?	**20.** There are 7 pencils in the drawer. 3 of the pencils are sharp and the rest are dull. How many of the pencils are dull?

Common Core CGI



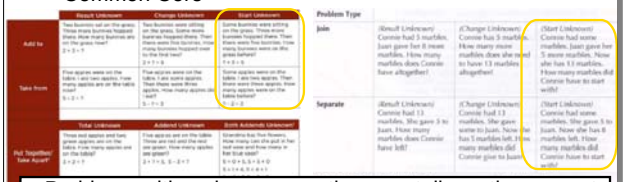
Problems with a change over time can tell you the amount you start with and the amount it changes and ask for the **Result (Result Unknown)**

Common Core CGI



Problems with a change over time can tell you the amount you start with and the amount you end with and ask for the amount of the **Change (Change Unknown)**

Common Core CGI



Problems with a change over time can tell you the amount of the change and the amount you end with and ask for the amount at the **Start (Start Unknown)**

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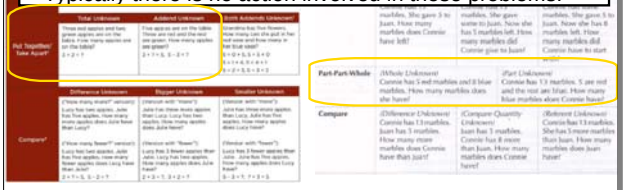

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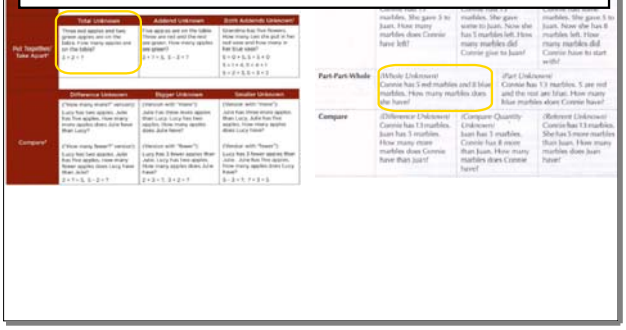
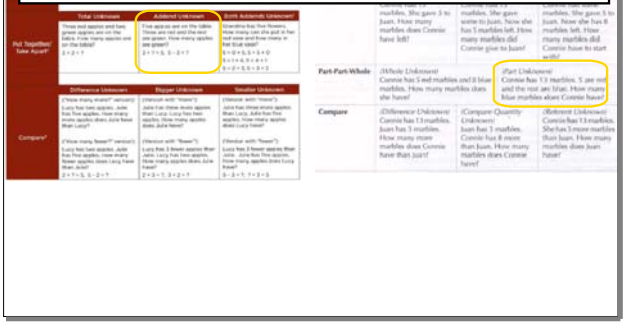
In a **Whole Unknown** or a **Total Unknown** problem, you know the sizes of the smaller parts, and you are asked to find the size of the larger set (that is made up of the parts).



In an **Addend Unknown** or a **Part Unknown** problem, you know the size of one (or more) of the smaller parts and you know the size of the whole/total, and you are asked to find the size of the unknown remaining part.



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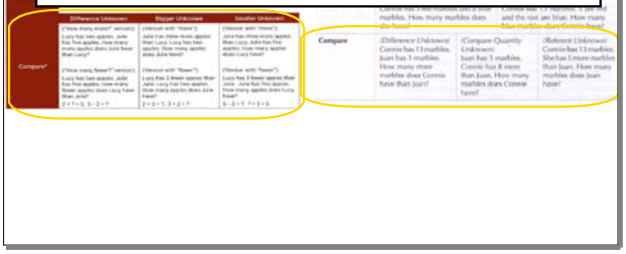

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In **Difference Unknown** problems, the amounts of the sets being compared is known, and the problem asks **how many more** or **how many fewer** are in one group than the other.

Common core breaks up the remaining compare problems by whether the unknown is the **bigger** or the **smaller** of the sets being compared. This corresponds to whether the natural operation to use to solve the problem is addition or subtraction

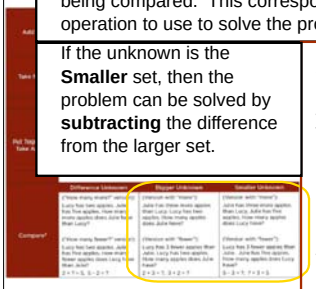
If the unknown is the B**igger** set, then the problem can be solved by **adding** the difference to the smaller set.

Bigger Unknown
(Version with "more"):
Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?

(Version with "fewer"):
Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?
2+3=? 3+2 =?

Common core breaks up the remaining compare problems by whether the unknown is the **bigger** or the **smaller** of the sets being compared. This corresponds to whether the natural operation to use to solve the problem is addition or subtraction

If the unknown is the **Smaller** set, then the problem can be solved by **subtracting** the difference from the larger set.



Smaller Unknown
(Version with "more"):
Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?

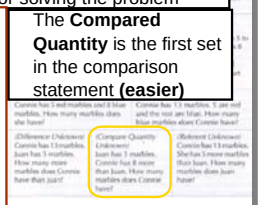
(Version with "fewer"):
Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?
5-3=? ?+3=5

CGI breaks up the remaining compare problems by whether the the problem is easier or harder for children to understand and figure out how to solve.
This corresponds to how the sentences are worded, which corresponds to whether the action implied by the comparison words matches the natural operation for solving the problem

Compared Quantity Unknown
(Version with "~~more~~"):
~~Julie has~~ three more apples than Lucy. Lucy has two apples. How many apples does ~~Julie have~~?
2+3=? 3+2 =?

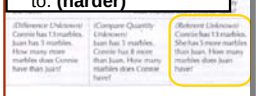
(Version with "~~fewer~~"):
~~Lucy has~~ 3 fewer apples than Julie. Julie has five apples. How many apples does ~~Lucy~~ have?
5-3=? ?+3=5

The **Compared Quantity** is the first set in the comparison statement **(easier)**



CGI breaks up the remaining compare problems by whether the the problem is easier or harder for children to understand and figure out how to solve.
This corresponds to how the sentences are worded, which corresponds to whether the action implied by the comparison words matches the natural operation for solving the problem

Referent Unknown
(Version with "~~more~~"):
Julie has three more apples ~~than Lucy~~. Julie has five apples. How many apples does ~~Lucy~~ have?
5+3=? ?+3=5

(Version with "~~fewer~~"):
Lucy has 3 fewer apples ~~than Julie~~. Lucy has two apples. How many apples does ~~Julie~~ have?
2+3=? 3+2 =?

The **Referent** is the second set in the comparison statement: the set being compared to. **(harder)**