5.) Can we get every possible line through the intersection point by using linear combinations of R_1 and R_2 for the specific example we did in class?

$$\begin{cases} 2x + y = 1 \\ x - y = 5 \end{cases}$$

$a * R_1 + b * R_2$ *This is a generic linear combination*

$a(2x + y) + b(x - y) = a + 5b$

$(2a + b)x + (a - b)y = (a + 5b)$

$y = -\frac{2a+b}{a-b}x + (a + 5b)$

$\therefore$ slope $= m = -\frac{2a+b}{a-b} = \frac{2a+b}{b-a}$ *Note:* $a, b \in \mathbb{R}$; at least one of a or $b \neq 0$; $a \neq b$

We don't want to let both a and b be 0 or we get 0=0. If a=b then the slope m is undefined. If you let a=b and simplify the linear combination equation you get x=2 which is the vertical line through the point of intersection

We will verify that $m = \frac{2a+b}{b-a}$ can take on any real number by showing that $2a + b$ and $b - a$ are independent of each other.

If we let c be any constant and let $b - a = c$, then $b = a + c$

By substitution $m = \frac{2a+b}{b-a} = \frac{2a+a+c}{c} = \frac{3a+c}{c} = \frac{3a}{c} + 1.$

For instance, if you let c=3, then you get m=a+1 and then it's easy to see that you can get any real number by choosing the right value for a.

This shows that denominator $b - a$ is independent of the numerator $2a + b$ allowing us to verify that $m = \frac{2a+b}{b-a}$ can take on any real number, furthermore verifying that we can get every possible line through the intersection point (2, -3).