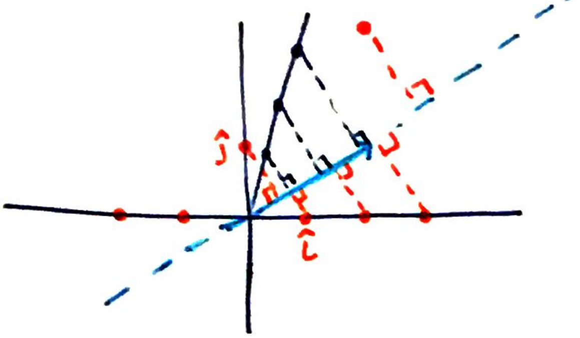
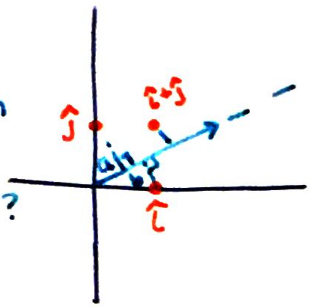
Projections



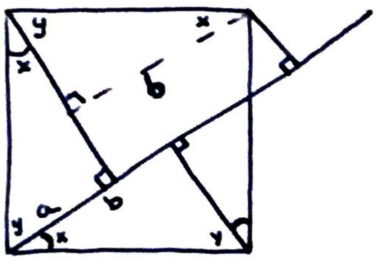
- projecting evenly spaced points will stay evenly spaced.

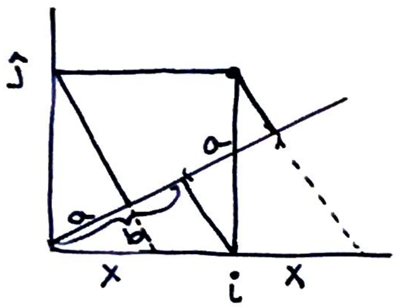
- Is the projection of the sum the sum of the projections?

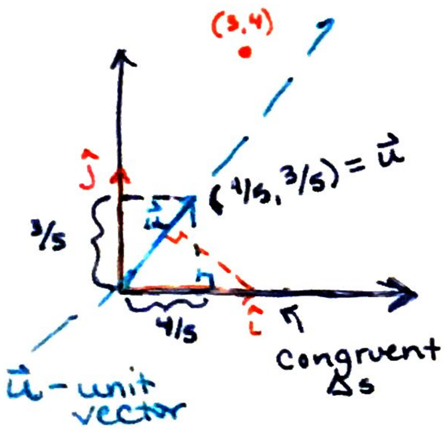
Yes



Why is the projection of $\hat{\imath} + \hat{\jmath} = a + b$?







What is the length of the projection of a vector?

The length of projection of $\hat{\imath}$ onto $(4/5, 3/5)$ is $4/5$.
$\hat{\jmath}$ onto $(4/5, 3/5)$ is $3/5$.

$$(1,0) \cdot (4/5, 3/5) = 4/5 + 0 = 4/5$$
$$(0,1) \cdot (4/5, 3/5) = 0 + 3/5 = 3/5$$

$(3,4) = 3\hat{\imath} + 4\hat{\jmath}$ → project onto $\vec{u}$

$3(4/5) + 4(3/5) = 24/5$ same as dot product

$$(3,4) \cdot (4/5, 3/5) = \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 4/5 \\ 3/5 \end{bmatrix}$$

Project onto $2\vec{u}$?

You would project then scale.

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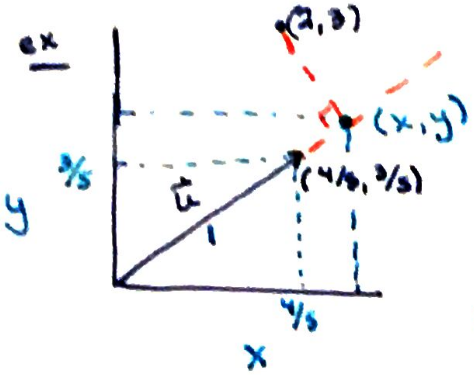
ex



$(2,3)\cdot(4/5, 3/5) = 2\cdot 4/5 + 3\cdot 3/5$

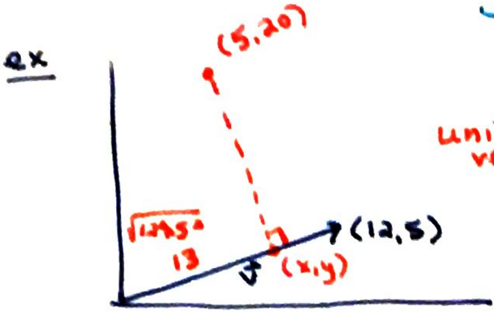
$8/5 + 9/5 = 17/5$

The length $= 17/5$

$x = 17/5 \cdot 4/5 = 68/25$ $\quad(68/25, 51/25)$

$y = 17/5 \; 3/5 = 51/25$

ex

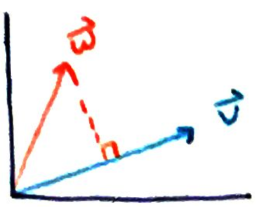


unit vector $\rightarrow (12/13, 5/13)\cdot(5,20)$

$60/13 + 100/13 = 160/13$ = length

$x = 160/13 \cdot 12/13 = 1920/169$ $\quad\left(\frac{1920}{169}, \frac{800}{169}\right)$

$y = 160/13 \cdot 5/13 = 800/169$



Project $\vec{w}$ onto $\vec{v}$: $\text{proj}_v(w)$

$$\text{length}\rightarrow\Big(\underbrace{\Big(\frac{\vec{v}}{|\vec{v}|}\Big)\cdot\vec{w}}_{\text{scaler}}\Big)\frac{\vec{v}}{|\vec{v}|}$$

same as $\left(\dfrac{\vec{v}\cdot\vec{w}}{|\vec{v}|}\right)\dfrac{\vec{v}}{|\vec{v}|}$

$$= \frac{(\vec{v}\cdot\vec{w})\,\vec{v}}{|\vec{v}|^2} = \frac{(\vec{v}\cdot\vec{w})\,\vec{v}}{(\vec{v}\cdot\vec{v})}$$

ex $|\vec{v}|^2 = \sqrt{12^2+5^2}^{\,2}$

$\vec{v}^{\,2} = 12^2 + 5^2$

$= 12\cdot 12 + 5\cdot 5 = (\vec{v}\cdot\vec{v})$