

The 3 row operations affect the determinate in 3 ways:

① Flipping an adjacent row negates the original determinate matrix (B)

$$\begin{array}{c}
 \begin{matrix} A & B & (A \times B) \end{matrix} \\
 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 1 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \\
 \text{Det } A \quad \cdot \quad \text{Det } B \quad = \quad \text{Det}(A \times B) \\
 -1 \quad \quad \quad 1 \quad \quad \quad -1
 \end{array}$$

② When multiplying a matrix by a scalar the original determinat matrix (B) is also multiplied by that scalar

$$\begin{array}{c}
 \begin{matrix} A & B & (A \times B) \end{matrix} \\
 \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 1 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 10 & 5 \\ 1 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} \\
 \text{Det } A \quad \cdot \quad \text{Det } B \quad = \quad \text{Det}(A \times B) \\
 5 \quad \quad \quad 1 \quad \quad \quad 5
 \end{array}$$

③ When a linear combination of the rows is applied to the matrix no change to the determinate happens

$$\begin{array}{c}
 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 1 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 1 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} \quad R_2 + R_1 \rightarrow \\
 \text{Det } A \quad \cdot \quad \text{Det } B \quad = \quad \text{Det}(A \times B) \\
 1 \quad \quad \quad 1 \quad \quad \quad 1
 \end{array}$$

You can use Gaussian Elimination to find the determinate of a matrix

example $\begin{bmatrix} 0 & 2 & 1 \\ 1 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} = A$ the determinate should end up being 1 (by graphing calc)

start with $\textcircled{1} \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ then $R_1 + R_3 \rightarrow R_3$ $\textcircled{2} \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 1 \\ 0 & -1 & -1 \end{bmatrix}$
 switching R_1 & R_2 this makes $\text{Det } A$ multiply (-1) linear combo has no effect on the determinate

next multiply $\frac{1}{2}$ to R_2 making det $\frac{1}{2}$ of original $\textcircled{3} \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & \frac{1}{2} \\ 0 & -1 & -1 \end{bmatrix}$ then $R_2 + R_3 \rightarrow R_3$ $\textcircled{4} \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$
 again linear combo has no effect on Det

at this point a triangle of zero's is made so the diagonal is the altered matrix determinate

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

Altered matrix $\text{det} = 1 \cdot 1 \cdot -\frac{1}{2} = -\frac{1}{2}$

Therefore

$$\text{Det } A \cdot \overset{\text{step 1 change}}{\downarrow} (-1) \cdot \overset{\text{step 3 change}}{\uparrow} \left(\frac{1}{2}\right) = \overset{\text{altered}}{\downarrow} -\frac{1}{2} \text{ matrix Det}$$

§ $\text{Det } A = -\frac{1}{2}(-1)(2) = 1 \quad \text{☺}$