Tuesday 7-30-19 (pm)

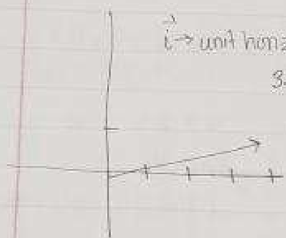
$\mathbb{R}^2$ is a two dimensional vector space

vector - magnitude + direction

$\vec{i}$ → unit horizontal $\vec{j}$ → vert component

$3.5i + j$

$\langle 3.5, 1 \rangle$ $\begin{bmatrix} 3.5 \\ 1 \end{bmatrix}$

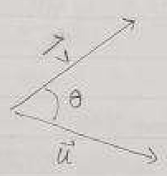


$\vec{u}$ and $\vec{v}$ are vectors
- add vectors
- multiply by scalar $a \cdot \vec{u} = \vec{u} \cdot a$
- dot product $\vec{u} \cdot \vec{v}$ → scalar

(inner product space)

$\vec{u} \cdot \vec{u} = |u|^2$ magnitude

cross product only found in $\mathbb{R}^3$



$\vec{u} \cdot \vec{v} = 0$ then $\vec{u} \perp \vec{v}$

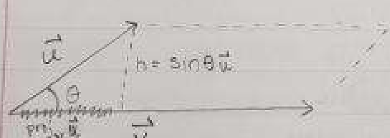
$\vec{u} \cdot \vec{v} = \cos\theta \, |\vec{u}| \, |\vec{v}|$

$\langle 2, 3 \rangle \cdot \langle 4, 6 \rangle$

$2 \cdot 4 + 3 \cdot 6 = \underbrace{26}$

$8 + 18$

$\sin\theta = \frac{h}{\vec{u}}$



$\langle x_1, y_1 \rangle \cdot \langle x_2, y \rangle$

dot product $= x_1 x_2 + y_1 y_2$

(*) area $= \vec{u} \vec{v} \sin\theta$

→ $\vec{i}$ and $\vec{j}$ form a basis for $\mathbb{R}^2$

($\perp$) orthogonal basis

linear combination of $\vec{i}$ and $\vec{j}$ → spanning set

describes any vector in space $\mathbb{R}^2$

$\vec{v} = a\vec{i} + b\vec{j}$ and $a = b = 0$

identity vector

basis for $\mathbb{R}^2$

Yes ☺	No ☹
$\{ \langle 1,0 \rangle, \langle 0,1 \rangle \}$	$\{ \langle 1,0 \rangle \}$
$\{ \langle 3,0 \rangle, \langle 0,\frac{1}{2} \rangle \}$	$\{ \langle 1,0 \rangle, \langle 0,1 \rangle, \langle 1,1 \rangle \}$
$\{ \langle 1,1 \rangle, \langle 1,0 \rangle \}$	$\{ \langle 1,2 \rangle, \langle 2,4 \rangle \}$
$\{ \langle 1,2 \rangle, \langle 1,-1 \rangle \}$	$\{ \langle 0,0 \rangle, \langle 1,2 \rangle, \langle 1,0 \rangle \}$

I want to get $\langle 2,3\rangle$ as a linear combination of the basis vectors

$\{\langle 1,1\rangle, \langle 2,1\rangle\}$

$a\langle 1,1\rangle + b\langle 2,1\rangle = \langle 2,3\rangle$

$a + 2b = 2$

$a + b = 3$

$\boxed{b = -1 \quad a = 4}$

$4\langle 1,1\rangle - \langle 2,1\rangle = \langle 2,3\rangle$

(one solution)

basis 1 and only 1 solution for every vector in $\mathbb{R}^2$

→ Every vector in $\mathbb{R}^2$ has an unique representation as a linear combination of basis vectors
- span
- linearly independent

Big Idea "What is a Basis"

$\{\langle 1,0\rangle, \langle 1,1\rangle, \langle 1,2\rangle\}$

$a\langle 1,0\rangle + b\langle 1,1\rangle + c\langle 1,2\rangle = \langle 2,3\rangle$

$a + b + c = 2$

$b + 2c = 3$

$\boxed{b = 3-2c}$

$a + 3 - 2c + c = 2$

$a - c = -1 \quad \boxed{a = c-1}$

infinite solutions $\quad$ $\boxed{c \text{ is any value}}$

- some vectors have multiple representations (wrong)

$a\langle 1,0\rangle + b\langle 1,1\rangle = \langle 1,2\rangle$ then $\langle 1,0\rangle$ and $\langle 1,1\rangle$ are linearly dependent (can create one of vectors from other two)

$\{\langle 1,2\rangle, \langle 2,4\rangle\}$

$a + 2b = 2$

$2a + 4b = 3$

parallel vectors

No Sol

only get scalar multiples of vectors → don't get all vectors (wrong - doesn't span)

Basis for $\mathbb{R}^3$? If not why not

$\{ \langle 1,2,3 \rangle, \langle 4,5,6 \rangle, \langle 7,8,9 \rangle \}$

Let's try to $\langle 1,0,1 \rangle$

$$\left|\begin{array}{ccc|c} 1 & 4 & 7 & 1 \\ 2 & 5 & 8 & 0 \\ 3 & 6 & 9 & 1 \end{array}\right| \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

No sol   no sol

Not Basis

Determinant is zero

some vectors are not unique

and some vectors are not made $\langle 1,0,1 \rangle$

$\therefore$ does not span and not linearily independent $\langle 3,3,3 \rangle$

$\{ \langle 2,1,0 \rangle, \langle 0,1,1 \rangle, \langle 2,4,2 \rangle \}$

$$\left|\begin{array}{ccc|c} 2 & 0 & 2 & 1 \\ 1 & 1 & 4 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right| \Rightarrow \begin{bmatrix} 2 \\ 4 \\ -1.5 \end{bmatrix}$$

one unique solution

so is a

basis

span, linearly independent

determinant $\neq 0$

Homework pg 178-9 1 a,d, 5, 21, 25, 45