Start of Proof of Cofactor (Cramer's) Method of finding Inverse

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \rightarrow \begin{bmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & -\begin{vmatrix} d & f \\ g & i \end{vmatrix} & \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ -\begin{vmatrix} b & c \\ h & i \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} & -\begin{vmatrix} a & b \\ g & h \end{vmatrix} \\ \begin{vmatrix} b & c \\ e & f \end{vmatrix} & -\begin{vmatrix} a & c \\ d & f \end{vmatrix} & \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix} \xrightarrow{\text{transpose}} \overset{A^{-1}}{\frac{1}{\det(A)} \begin{bmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & -\begin{vmatrix} b & c \\ h & i \end{vmatrix} & \begin{vmatrix} b & c \\ e & f \end{vmatrix} \\ -\begin{vmatrix} d & f \\ g & i \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} & -\begin{vmatrix} a & c \\ d & f \end{vmatrix} \\ \begin{vmatrix} d & e \\ g & h \end{vmatrix} & -\begin{vmatrix} a & b \\ g & h \end{vmatrix} & \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix}}$$

If A^{-1} is truly the inverse then $A^{-1} \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\det A$ (using column 1) $= a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - d\begin{vmatrix} b & c \\ h & i \end{vmatrix} + g\begin{vmatrix} b & c \\ e & f \end{vmatrix}$ to prove try each value

can be found using any row/colum (choose wisely) :)

$$\frac{1}{\det A} \begin{bmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & -\begin{vmatrix} b & c \\ h & i \end{vmatrix} & \begin{vmatrix} b & c \\ e & f \end{vmatrix} \\ -\begin{vmatrix} d & f \\ g & i \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} & -\begin{vmatrix} a & c \\ d & f \end{vmatrix} \\ \begin{vmatrix} d & e \\ g & h \end{vmatrix} & -\begin{vmatrix} a & b \\ g & h \end{vmatrix} & \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} x=1 & 0 & 0 \\ y=0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = \frac{1}{\det A}\Bigg(\underbrace{a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - d\begin{vmatrix} b & c \\ h & i \end{vmatrix} + g\begin{vmatrix} b & c \\ e & f \end{vmatrix}}_{\text{this is a determinant and happens to be det A}}\Bigg)$$

but the $\det(A) \overset{\checkmark}{=} 1$

$$y = \frac{1}{\det A}\Bigg(\underbrace{-a\begin{vmatrix} d & f \\ g & i \end{vmatrix} + d\begin{vmatrix} a & c \\ g & i \end{vmatrix} - g\begin{vmatrix} a & c \\ d & f \end{vmatrix}}\Bigg)$$

since always in pairs then they are the columns
$\begin{matrix} a & c \\ d & f \\ g & i \end{matrix}$
(since $-a, d, -g$ then middle column)

this is also a determinant... but not of A → lets find the matrix it is the determinant of and call it B

↓ coeffs

If det B = ↑ then B would have to be

$$B = \begin{bmatrix} a & a & c \\ d & d & f \\ g & g & i \end{bmatrix}$$

Since two columns (or two rows of B) are the same we know that $\det B = 0$ ∴ $y = \frac{1}{\det A} \cdot \det B \overset{\checkmark}{=} 0$

Could repeat this for every value in the identy matrix