Ex. Using RREF to find inverse matrix.

Goal: Put this matrix in RREF ↗

Remember to multiply on left to do row operations. Do it on both sides to keep equal.

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cancel{\;\;} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cancel{\;\;}$$

You can get rid of these.

This elementary matrix switches the rows →

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

OR $\begin{bmatrix} 2 & 3 & | & 1 & 0 \\ 1 & -1 & | & 0 & 1 \end{bmatrix}$

Switch rows

$$\begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\begin{bmatrix} 1 & -1 & | & 0 & 1 \\ 2 & 3 & | & 1 & 0 \end{bmatrix}$

$-2R_1 + R_2 \rightarrow R_2$

Changes this to be zero →

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

$\begin{bmatrix} 1 & -1 & | & 0 & 1 \\ 0 & 5 & | & 1 & -2 \end{bmatrix}$

$\tfrac{1}{5} R_2 \rightarrow R_2$

Changes this to be one →

$$\begin{bmatrix} 1 & 0 \\ 0 & \tfrac{1}{5} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \tfrac{1}{5} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \tfrac{1}{5} & -\tfrac{2}{5} \end{bmatrix}$$

$\begin{bmatrix} 1 & -1 & | & 0 & 1 \\ 0 & 1 & | & \tfrac{1}{5} & -\tfrac{2}{5} \end{bmatrix}$

$R_2 + R_1 \rightarrow R_1$

Changes this to be zero →

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \tfrac{1}{5} & -\tfrac{2}{5} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \tfrac{1}{5} & \tfrac{3}{5} \\ \tfrac{1}{5} & -\tfrac{2}{5} \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & | & \tfrac{1}{5} & \tfrac{3}{5} \\ 0 & 1 & | & \tfrac{1}{5} & -\tfrac{2}{5} \end{bmatrix}$

↓ identity matrix

↓ inverse matrix

Notice:

Matrix	Row Operation
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	Switch rows
$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$	$-2R_1 + R_2 \rightarrow R_2$
$\begin{bmatrix} 1 & 0 \\ 0 & \tfrac{1}{5} \end{bmatrix}$	$\tfrac{1}{5} R_2 \rightarrow R_2$
$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$	$R_2 + R_1 \rightarrow R_1$

* Start w/ identity matrix and change to what you want in the spot you want