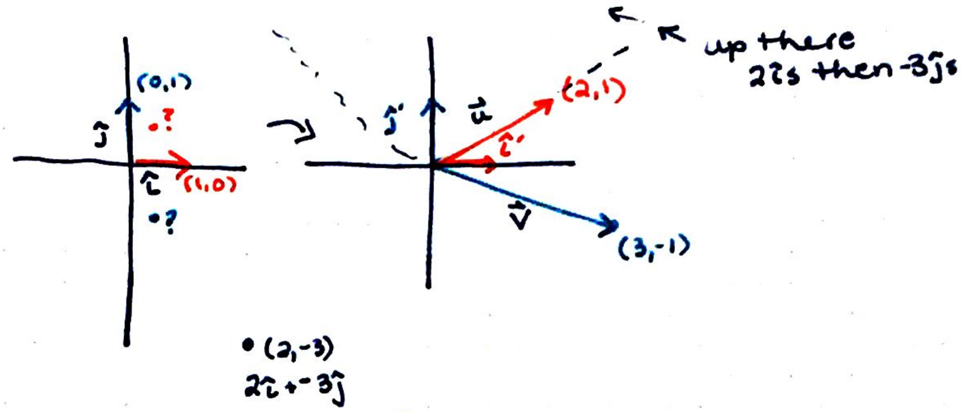
$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$



$\{\vec{u}, \vec{v}\}$ is a basis for $\mathbb{R}^2$

$a \cdot \vec{u} + b\vec{v} = \hat{\imath}'$ ← can get to $\hat{\imath}'$ using $\vec{u} + \vec{v}$ $+\hat{\jmath}'$

$c\vec{u} + d\vec{v} = \hat{\jmath}'$

$a\begin{bmatrix} 2 \\ 1 \end{bmatrix} + b\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$c\begin{bmatrix} 2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 1 & -1 & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 1 & -1 & 1 \end{array}\right]$

how to get these

$$\begin{array}{cccc} a/c & b/d & \hat{\imath}' & \hat{\jmath}' \\ \end{array}$$
$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1/5 & 3/5 \\ 0 & 1 & 1/5 & -2/5 \end{array}\right]$$
$$\begin{array}{cc} a & c \\ b & d \end{array}$$

$\frac{1}{5}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \frac{1}{5}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

point is $(1/5, 1/5)$
to get to $(1,0)$

$\frac{3}{5}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + -\frac{2}{5}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

point is $(3/5, -2/5)$
to get to $(0,1)$

inverse is $\begin{bmatrix} 1/5 & 3/5 \\ 1/5 & -2/5 \end{bmatrix}$