

$\{(1,1,1,1), (2,5,2,5), (1,2,3,0)\}$  these are independent, do they span  $\mathbb{R}^4$ ?

Representative vectors in  $\mathbb{R}^4$

$$\langle 1, 0, 0, 0 \rangle$$

$$\langle 0, 1, 0, 0 \rangle$$

$$\langle 0, 0, 1, 0 \rangle$$

$$\langle 0, 0, 0, 1 \rangle$$

$$\langle x, y, z, w \rangle$$

$$a(1,1,1,1) + b(2,5,2,5) + c(1,2,3,0) = (x, y, z, w)$$

$$\begin{bmatrix} 1 & 2 & 1 & | & x \\ 1 & 5 & 2 & | & y \\ 1 & 2 & 3 & | & z \\ 1 & 5 & 0 & | & w \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & | & x \\ 0 & 3 & 1 & | & y-x \\ 0 & 0 & 2 & | & z-x \\ 0 & 3 & -1 & | & w-x \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & | & x \\ 0 & 3 & 1 & | & y-x \\ 0 & 0 & 2 & | & z-x \\ 0 & 0 & -2 & | & w-y \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & | & x \\ 0 & 3 & 1 & | & y-x \\ 0 & 0 & 2 & | & z-x \\ 0 & 0 & 0 & | & w-y+z-x \end{bmatrix}$$

not always 0, so it does not span.

$$w-y+z-x = 0 \text{ OR}$$

$$(2, 3, 2, 3) \text{ OR } (1, 1, 0, 2)$$

$x \quad y \quad z \quad w$

$$\begin{bmatrix} 1 & 2 & 1 & | & 2 \\ 1 & 5 & 2 & | & 3 \\ 1 & 2 & 3 & | & 2 \\ 1 & 5 & 0 & | & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 4/3 \\ 0 & 1 & 0 & | & 1/3 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$a = 4/3$$

$$b = 1/3$$

$$c = 0$$

$$d = 0$$

$$w-y+z-x \neq 0 \text{ OR}$$

$$(1, 2, 3, 4) \text{ OR } (1, 0, 0, 0)$$

$x \quad y \quad z \quad w$

$$\begin{bmatrix} 1 & 2 & 1 & | & 1 \\ 1 & 5 & 2 & | & 2 \\ 1 & 2 & 3 & | & 3 \\ 1 & 5 & 0 & | & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

$$a = 0$$

$$b = 0$$

$$c = 0$$

$$d = 1$$