7/31/19 Math: 751

Testing for Linear Independence / Dependence use Matrices

(Please reference pages 173 - 176 in Linear Algebra text for more information!)

Testing for Linear Independence / Dependence

Let $S = \{V_1, V_2, \ldots, V_k\}$ be a set of vectors in a vector space V. Use the following steps:

Given

$S_1 = \{v_1, v_2, v_3\} = \{\langle 1,2,3\rangle, \langle 0,1,2\rangle, \langle -2,0,1\rangle\}$

1) Write a sytem of equations from the vector equation

vector equation: $a\langle 1,2,3\rangle + b\langle 0,1,2\rangle + c\langle -2,0,1\rangle = \langle 0,0,0\rangle$

(this is testing for a non-trivial solution)

$$\begin{aligned} 1a + 0b - 2c &= 0 \\ 2a + 1b + 0c &= 0 \\ 3a + 2b + 1c &= 0 \end{aligned}$$

2) Put into an augmented matrix and use Gaussing elimination to determine if there is a solution besides the trivial one of $a=0, b=0, c=0$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \end{array}\right] \xrightarrow[\text{of row Operations!}]{\text{do a bunch}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Since we only get the trivial solution of $a=b=c=0$, the vectors of S_1 are <u>linearly independent</u>

continued...

$\boxed{\text{Linearly Dependent example:}}$

$S_2 = \{\langle 1,2,-1,3\rangle, \langle 2,1,4,0\rangle, \langle 11,10,13,9\rangle\}$

vector eqn: $a\langle 1,2,-1,3\rangle + b\langle 2,1,4,0\rangle + c\langle 11,10,13,9\rangle = \langle 0,0,0,0\rangle$

gives us:

$$\begin{aligned} a + 2b + 11c &= 0 \\ 2a + b + 10c &= 0 \\ -a + 4b + 13c &= 0 \\ 3a \quad + 9c &= 0 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 11 & 0 \\ 2 & 1 & 10 & 0 \\ -1 & 4 & 13 & 0 \\ 3 & 0 & 9 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \rightarrow a + 3c = 0 \\ \rightarrow b + 4c = 0 \\ \rightarrow 0 = 0 \\ \rightarrow 0 = 0 \end{array}$$

b depends on c!

In this situation, one vector is a linear combination of the other two.

One combination: if $c = 1$

$a + 3c = 0 \rightarrow a + 3 = 0 \rightarrow a = -3$

$b + 4c = 0 \rightarrow b + 4 = 0 \quad b = -4$

so $-3\langle 1,2,-1,3\rangle - 4\langle 2,1,4,0\rangle = \langle 11,10,13,9\rangle$

Possible rref augmented matrices and what they mean!

a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ → only the trivial solution $\boxed{\text{Independent}}$

b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ → only the trivial solution $\boxed{\text{Independent}}$

c) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ → has a non-trivial solution $\boxed{\text{Dependent}}$

c) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ non-trivial solution $\boxed{\text{Dependent}}$

d) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\boxed{\text{Dependent}}$