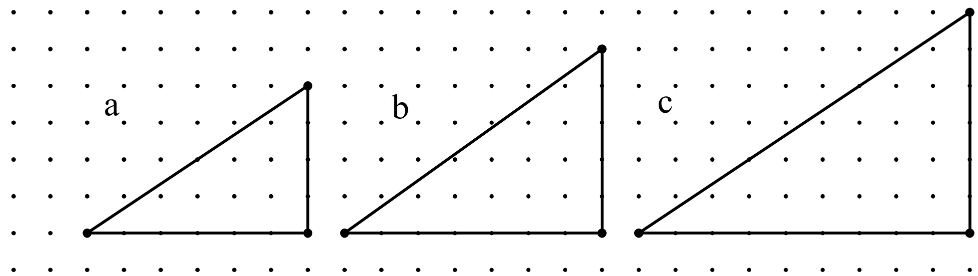
Homework on similarity, geometric series, and the size of a self-similar fractal.

**Similarity problems:**

1. Are all of these triangle similar? Are any of these triangles similar? Tell how you know:




2. Jack climbed up the bean stalk, and went into the giant's house. The giant's height is 4x Jack's height. If everything in the giant's house is proportional (geometrically similar) to things in Jack's house, answer the following questions:

a. If Jack's tabletop is 8 square feet, what is the area of the giant's table top?

b. If the circumference of Jack's plate is 2 ft, what is the circumference of the giant's plate?

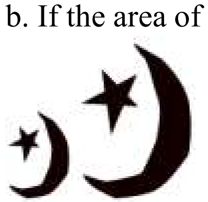
c. If Jack's mug holds 1 cup of water, how much water can the giant's mug hold?

d. If Jack weighs 100 lbs, how much does the giant weigh (assume Jack and the giant are also similar in shape, and have the same density)?

3. I have two similar (proportional) pictures of a moon and star. The larger picture was enlarged to 200% on a standard copy machine from the smaller one:

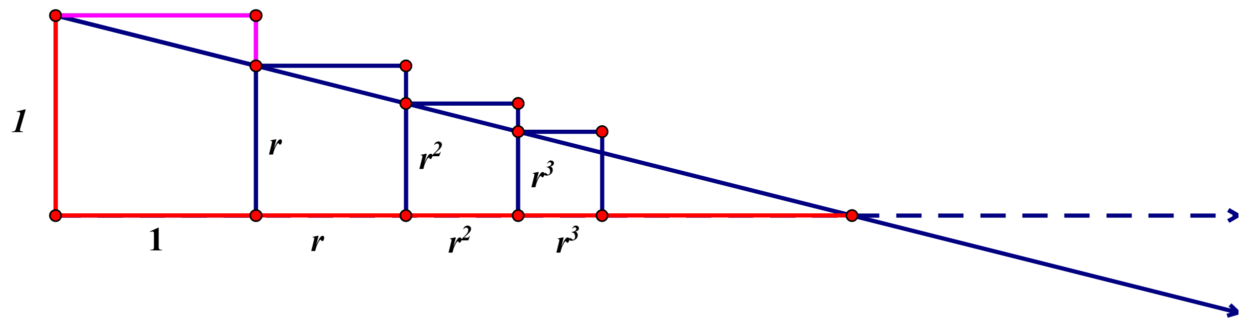
a. If the area of the smaller star is 5 $cm^2$, what is the area of the larger star?

b. If the area of the larger moon is 48 $cm^2$, what is the area of the smaller moon?



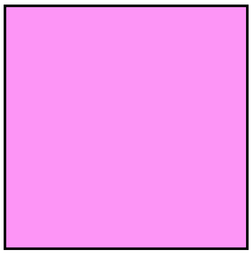
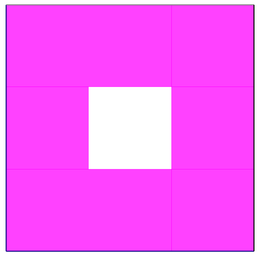
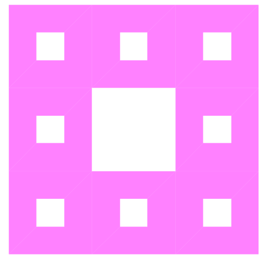
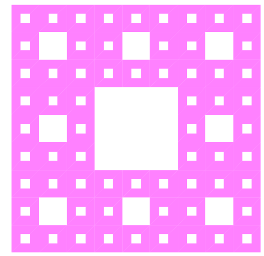
**Geometric series problem:**

4. Explain the proof of the geometric series formula illustrated by this diagram by explaining:
a. How do we know that one straight line goes through all of the top-left corners of the squares?
b. What two similar triangles are we using? How do we know they are similar (hint: use angles)?
c. What similar triangle ratio can we use to find the sum of the infinite series: $1 + r + r^2 + r^3 + \ldots$



**Self-similar fractal problems:**

5. A square version of a Sierpinski triangle is a Sierpinski carpet:
Rule: subdivide each square into 9 smaller squares and erase the middle one.

| Showing the area<br>Iteration 0:<br> | Iteration 1:<br> | Iteration 2:<br> | Iteration 3:<br> |
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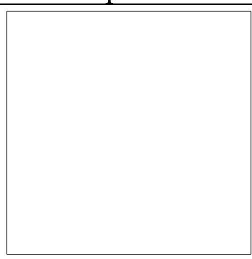
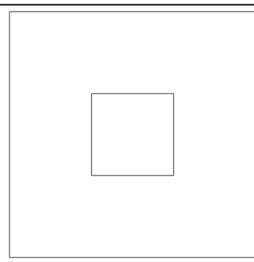
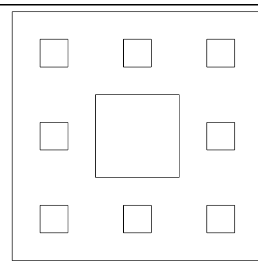
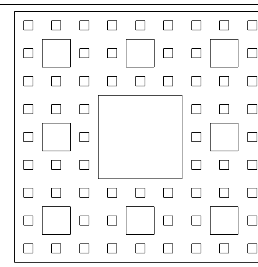
| Just the perimeter | | | |
|  |  |  |  |

a. Find the formula for the total area of an iteration *n* Sierpinski carpet. Use the formula to show that in the limit the area is 0. You should assume that the area of an iteration 0 carpet is 1 S (S for square area), and make a helpful table.

b. Find the formula for the total length of the "perimeter" of an iteration *n* Sierpinski carpet. Use the formula to show that the limit of the length is infinite. You should assume that the length of a side of the iteration 0 square is 1 u (u is for unit), make a helpful table.

Note: this one is more complicated than the Sierpinski triangle, and I had to use the geometric series formula to get a formula. The "4" from the iteration 0 square doesn't really fit nicely with the rest of it, so your formula will (unless you are more clever than me) be 4 + (geometric series). Also the pattern probably won't be clear until you have something written down for each of the iterations 1-3.

Note the finite geometric series formula (which you will need to use for the n-th term is:

$S = \dfrac{a - ar^n}{1 - r}$ where $a$ is the first term of the geometric series, $r$ is the common ratio and $n$ is the number of terms. Its limit is $\dfrac{a}{1-r}$ when $r < 1$ and its limit is infinite when $r > 1$