

An easy-puzzle example:

If you know that a polyhedron has faces that are :

- 8 triangles
- 2 squares
- 4 pentagons

How many edges and vertices does it have?

First find the number of faces: $8+2+4 = 14$ faces

Next find the number of edges on each face, and divide by 2 (because we are double counting total edges):

$$E = \frac{8\Delta + 2\square + 4[\text{pentagon}]}{2} = \frac{8 \cdot 3 + 2 \cdot 4 + 4 \cdot 5}{2} = \frac{52}{2} = 26$$

Now use the Euler characteristic of a polyhedron to say:

$$V - E + F = 2$$

$$V = E + 2 - F = 26 + 2 - 14 = 14$$