Concurrency Assignment 1: Prove each of these lemmas (easiest to hardest, I think)

Lemma 1: Given segment $\overline{AB}$, a point C lies on the perpendicular bisector of $\overline{AB}$ if and only if it is equidistant from points A and B.

Lemma 2: Given angle $\angle ABC$, a point D lies on the angle bisector of $\angle ABC$ if and only if the distance from D to $\overleftrightarrow{AB}$ is equal to the distance from D to $\overleftrightarrow{BC}$

Lemma 3: Given triangle ΔABC, if you construct lines parallel to each side going through the opposite vertex, that will create a large triangle that is split into four smaller triangles, one of which is ΔABC, and each of the smaller triangles are congruent to ΔABC. (Note: if you prove that ΔABC is congruent to one of the other three triangles using what you are given by the construction, you are allowed to say that "similarly" ΔABC is congruent to the other two without showing all of the steps)

Lemma 4a: Given triangle ΔABC, with point D on side $\overline{BC}$, then $\dfrac{area(\Delta ADC)}{area(\Delta ABC)} = \dfrac{DC}{BC}$

Lemma 4b: Given triangle ΔABC, with M being the midpoint of side $\overline{AC}$ and N being the midpoint of side $\overline{BC}$, and $P = \overline{BM} \cap \overline{AN}$ then $(PM) = \frac{1}{3}(BM)$

Hints on the next page. Waltkthrough videos online.

Hints

Lemma 1: Part 1: assume C is on the perpendicular bisector (which is the line which intersects $\overline{AB}$ at its midpoint, and is perpendicular to $\overline{AB}$). Draw in the segments from C to A and B.

Part 2: assume C has the same distance to A and B. Draw in the segments from C to A and B. Draw in the segment from C to the midpoint of $\overline{AB}$. Try to prove that segment is also perpendicular to $\overline{AB}$

Lemma 2:

- Part 1: assume D is on the angle bisector (which is the ray that splits the angle into two equal angles).
- Construct a segment from D to $\overleftrightarrow{BA}$ that is perpendicular to $\overleftrightarrow{BA}$, and a segment from D to $\overleftrightarrow{BC}$ that is perpendicular to $\overleftrightarrow{BC}$: the lengths of those segments are the distances from D to $\overleftrightarrow{BA}$ and $\overleftrightarrow{BC}$.
- Look for congruent triangles to help you prove that those segments have equal lengths.

- Part 2: Construct a segment from D to $\overleftrightarrow{BA}$ that is perpendicular to $\overleftrightarrow{BA}$, and a segment from D to $\overleftrightarrow{BC}$ that is perpendicular to $\overleftrightarrow{BC}$: the lengths of those segments are the distances from D to $\overleftrightarrow{BA}$ and $\overleftrightarrow{BC}$.
- You are given, in this part, that those lengths are equal.
- Draw in the ray $\overrightarrow{BD}$ or line $\overleftrightarrow{BD}$. try to prove that $\overrightarrow{BD}$ bisects the angle.

Lemma 3: When you draw your triangle ΔABC, make sure that you draw a scalene triangle: if you draw an isosceles triangle, it will be much harder to sort out what angles and sides are congruent.

Lemma 4a:

- Draw your triangle with $\overline{AC}$ on the bottom, and use it as the base when you calculate the areas of both triangles.
- You will need to draw in some altitudes to show the height (one for each triangle).
- Use those altitudes together with the segments $\overline{BC}$ and $\overline{DC}$ to make two new triangles. Those new triangles are similar triangles.
- Use similar triangle properties

Lemma 4b:

- Draw a scalene triangle
- Draw your triangle with $\overline{AC}$ on the bottom, and construct an altitude from B.
- Look for triangles that have equal areas
- Draw a segment from P to C

- Prove that the triangles ABM and MBC have equal areas (note: the triangles are not congruent)
- Draw an altitude from P to $\overleftrightarrow{AC}$
- Triangles APM and MPC also have equal areas
- Turn your page so it looks like $\overline{BC}$ is the base
- Draw an altitude from A to $\overleftrightarrow{BC}$
- Which triangles have equal areas when you look from this perspective?
- Draw another altitude from P
- Name some variables to be the areas of the small triangles that you see in your picture (there should be 5 of them)
- Write down some equations about equal areas.
- Can you find the ratio of areas between triangles APM and ABM?
- If you draw out just the part of your picture that has ABM and APM, does it look anything like lemma 4a?
- Use lemma 4a.