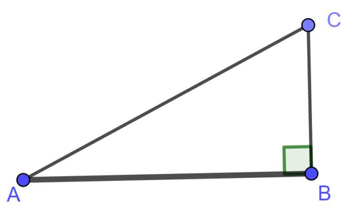
Geometry examples (what we did in class March 5th):

2. Show that a right triangle has area (1/2)bh by using rectangles

- Show that a right triangle whose base is one of the legs has area (1/2)bh by constructing a rectangle that contains the triangle (and shares two sides with the legs of the triangle)
- Prove that the rectangle consists of two congruent triangles (one of which is the original triangle.
- Note/show that the base and height of the triangle are the same as the length and width of the rectangle
- Note that the area of the triangle is half of the area of the rectangle (which is bh)

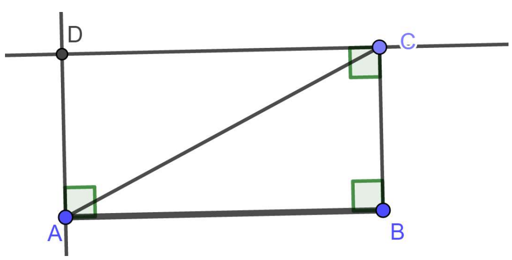
Start with a right triangle with base $\overline{AB}$ that is one of the legs of the triangle.

Note that then the length of the base is AB and the length of the height is BC



Construct a rectangle that contains it by:

- constructing a line perpendicular to $\overline{AB}$ at A, and
- a line perpendicular to $\overline{BC}$ at C.
- Name the intersection of these lines D



Because the sum of angles in a quadrilateral is 360° (*), and the angles of quadrilateral $ABCD$ at A, B, and C are 90°, we know that the angle at D is also 90°

Thus $ABCD$ is a rectangle, and so we know its area is $(AB)(BC)$

Also, because $ABCD$ is a rectangle, we know that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$ (*)

Thus $\Delta ABC \cong \Delta CDA$

That means $area\Delta ABC = area\Delta CDA$

We also know that $area\Delta ABC + area\Delta CDA = areaABCD = (AB)(BC)$

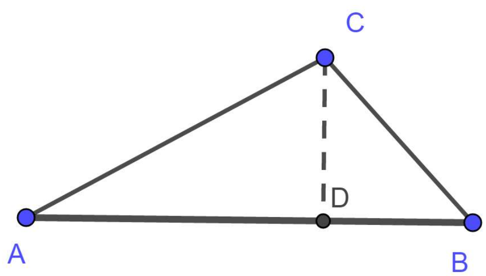
So (using algebra), we get that $area\Delta ABC = \frac{1}{2}(AB)(BC) = \frac{1}{2}bh$

(* these theorems will be added to the sheet)

3. Use (2) and subtraction to show that a triangle with a specific property has area $(1/2)bh$.

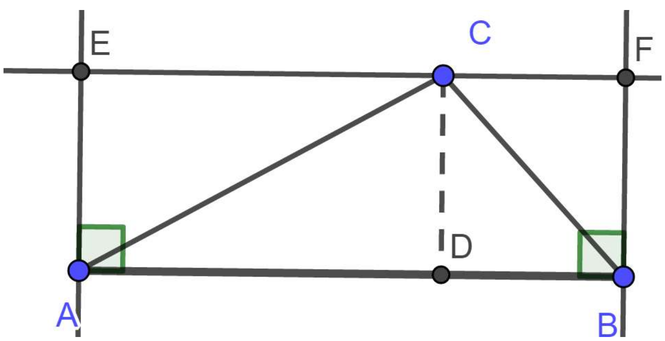
- Start with a triangle with an identified base where the altitude to that base lies inside the triangle.
- Construct a rectangle where one side is the base of the triangle, and where the opposite vertex lies on the opposite side of the rectangle.
- Note that the area of the rectangle consists of the original triangle area and the area of the two right triangles.
- Use algebra to write down (rectangle area)-(right triangle 1 area) – (right triangle 2 area) and simplify it to $(1/2)bh$.

To start, we have a triangle ΔABC with base $\overline{AB}$ that has length b, and altitude $\overline{CD}$ whose length h is the height.



Now construct a rectangle around it by constructing:

- a line perpendicular to $\overline{AB}$ through point A,
- a line perpendicular to $\overline{AB}$ through point B, and
- a line parallel to $\overline{AB}$ through point C
- name the intersection points E (adjacent to A) and F (adjacent to B)



Then the angles are E and F are right angles because of a parallel lines theorem (AIA or similar), so $ABFE$ is a rectangle.

Additionally, the altitude at C is perpendicular to $\overline{AB}$, and because of parallel lines, we also know that it is perpendicular to $\overline{EF}$.

Thus, $ADCE$ and $DBFC$ are rectangles, and so all of the side lengths BF, CD and AE are equal to the height h.

We know the area of rectangle $ABFE$ is bh,

Using the result from problem 2, we can calculate the area of right triangles, so we know:

$$area\Delta CEA = \frac{1}{2}(CE)h \quad \text{and we know} \quad area\Delta CFB = \frac{1}{2}(CF)h$$

We also know that the area of the rectangle is the sum of the areas of the three triangles:

$$areaABFD = area\Delta CEA + area\Delta CFB + area\Delta ABC$$

$$bh = \frac{1}{2}(CE)h + \frac{1}{2}(CF)h + area\Delta ABC$$

$$bh = \frac{1}{2}(CE + CF)h + area\Delta ABC$$

$$bh = \frac{1}{2}bh + area\Delta ABC$$

$$bh - \frac{1}{2}bh = area\Delta ABC$$

$$\frac{1}{2}bh = area\Delta ABC$$

Right here, we are again using that opposite sides of a rectangle have equal length so $CE + CF = EF = AB = b$